DdM | Didattica della matematica. Dalla ricerca alle pratiche d'aula
2026 (19), 10–38. https://doi.org/10.33683/ddm.26.19.1
OPEN ACCESS

Pragma-dialectics as a theoretical approach to argumentation in mathematics education

La pragma-dialettica come approccio teorico all'argomentazione in didattica della matematica

Lorenzo Cosci°•, Andrea Rocci• and Silvia Sbaragli°
°Dipartimento formazione e apprendimento / Alta scuola pedagogica, SUPSI – Svizzera
•Facoltà di comunicazione, cultura e società, USI – Svizzera

✉ lorenzo.cosci@supsi.ch, andrea.rocci@usi.ch, silvia.sbaragli@supsi.ch

Abstract / The practice of argumentation plays an extremely important role in countless contexts in the life of every individual, whether at work or in everyday life. Its importance is reflected in the educational field through the reference frameworks of international surveys and the guidelines of various countries, which highlight it as a tool to support learning and as a core skill in the education of every future citizen of a complex democratic society. This article proposes pragma-dialectical theory as an approach to argumentation in mathematics education, highlighting how this perspective is in line with national and international guidelines. Based on this theoretical approach, five essential analytical elements for pragma-dialectical argumentation are defined: verbalisation, awareness of reasons, dialogue, difference of opinion, and incentive for argumentation. Through these elements, some examples of argumentative tasks are analysed, highlighting how school practice sometimes considers as argumentative also teaching activities that exclude elements related to the pragmatic and social dimensions of argumentation emphasised by pragma-dialectical theory, which should instead be considered from an educational perspective.

Keywords: argumentation; pragma-dialectics; reasoning; task analysis.

Sunto / La pratica dell'argomentazione assume un ruolo di estrema rilevanza in innumerevoli contesti della vita di ciascun individuo, siano essi lavorativi o quotidiani. La sua importanza trova riscontro in ambito didattico, nei quadri di riferimento delle indagini internazionali e nelle indicazioni di diversi Paesi, in quanto strumento a supporto dell'apprendimento e come competenza centrale nella formazione di ogni futuro cittadino di una complessa società democratica. Questo articolo propone la teoria pragma-dialettica come approccio all'argomentazione in didattica della matematica, mettendo in evidenza come questa prospettiva risulti in linea con le indicazioni nazionali e internazionali. Sulla base di questo approccio teorico si definiscono cinque elementi analitici essenziali affinché l'argomentazione possa dirsi tale in senso pragma-dialettico: verbalizzazione, coscienza delle ragioni, dialogo, differenza di opinione e incentivo all'argomentazione. Attraverso questi elementi si analizzano alcuni esempi di quesiti argomentativi, evidenziando come la pratica scolastica consideri talvolta come argomentative attività didattiche che escludono elementi afferenti alle dimensioni pragmatica e sociale dell'argomentazione messe in luce dalla teoria pragma-dialettica, che invece sarebbero da considerare dal punto di vista didattico.

Parole chiave: argomentazione; pragma-dialettica; ragionamento; analisi di quesiti.

© 2026 Cosci Lorenzo, Rocci Andrea and Sbaragli Silvia. This is an Open Access text, published by Centro competenze didattica della matematica and Servizio risorse didattiche e scientifiche, eventi e comunicazione of DFA/ASP–SUPSI in collaboration with DECS. The text is distributed under the terms of the Creative Commons Attribution 4.0 International License (CC BY 4.0) that allows you to use, share and modify the text in any medium as long as the author and the original source are credited.

1 Introduction

Argumentation competence is of crucial importance in many areas of human existence, finding application in both professional challenges and everyday interactions. Argumentation comes into play whenever one does not merely state something or present information or data, but chooses to give reasons for one's position, seeking to convince someone regarding a certain topic or a decision to be made. We argue, for example, when we suggest a book to a friend or invite a colleague to lunch at a new restaurant. Argumentation plays a primary role in everyday, informal discussions and is at the same time the basis of democratic politics, as it serves a central function in the formally regulated contexts of legal and parliamentary debates.

The importance of argumentation in everyday life and in the civic sphere is inevitably reflected in the educational context as well. So much so that the various frameworks of international surveys, as well as the guidelines and school curricula of different countries, emphasize how the development of this competence is essential to the education of every student throughout the entire school career, from at least two perspectives: to support learning and to prepare students as future citizens of a complex democratic society.

As this article will show, to achieve this ambitious goal during the school years, it may be important to consider and adopt in the classroom the principles of the pragma-dialectical theory of argumentation, developed in Amsterdam by van Eemeren and Grootendorst between the 1970s and the late 1990s, and subsequently expanded by van Eemeren and Houtlosser in the following decades. This theory is grounded in the pragmatic, social, and dialectical aspects of argumentation, which are fundamental for effective interaction and debate in contemporary societies. Pragma-dialectics views argumentation as part of a discourse aimed at resolving a difference of opinion through the methodical verification of the validity of the claims stated by the interlocutors. In this approach, the relationship with the interlocutor plays a crucial role, as it is the interlocutor's critical attitude, understood as the need to find adequate reasons for actions, decisions, and beliefs, that contributes to the emergence and development of the argumentation itself.

These elements (pragmatic, social, and dialectical), which characterize argumentation from a pragma-dialectical perspective, should also be considered and emphasized in the teaching-learning process, particularly in mathematics. On the one hand, this is because knowledge construction takes place within the communicative relationship between teacher and students, and on the other, because teachers' didactical choices can be undertaken to treat disciplinary and cultural themes as catalysts for debates.

Currently, however, the most widespread school practice regarding argumentation in mathematics involves asking students to explain and describe the reasoning or problem-solving process of a task, without placing the activity within a dialectical and fully argumentative context in which different opinions are brought into question.

This article, therefore, aims to propose pragma-dialectical theory as a theoretical approach to argumentative practice in mathematics education. To do so, we will first show how the pragma-dialectical theory of argumentation is well-suited for developing argumentative skills in line with international and national guidelines. Next, after clarifying the characteristics of argumentation from a pragma-dialectical perspective, focusing on the relationship between reasoning and argumentation, we will analyse six mathematical tasks considered argumentative in school practice and the potential contexts in which they are embedded. These tasks differ significantly from one another in various respects: type, required linguistic act, and context in which they are implemented. The analysis aims to identify which and how many of the theoretical elements of argumentation from the pragma-dialectical perspective are stimulated. Considering the results obtained, we will demonstrate how, in some of these activities, elements related to the explicitation of reasoning predominate

over the characteristic elements of the pragma-dialectical perspective on argumentation. Finally, we highlight which didactical design elements can foster these analytical elements.
This analysis will lead to the conclusion that, for mathematics education to contribute to the development of argumentative skills for future citizens, as fostered by national and international educational guidelines, it is necessary for educational pathways to also include teaching practices in which argumentation serves as a tool for resolving a difference of opinion, embedded within meaningful contexts where all the elements of argumentation highlighted by pragma-dialectical theory are valued.

2 The role of argumentation in education

Educational researchers have recognized the practice of argumentation as fundamental from multiple perspectives: both as a competence to be developed by citizens and as a tool with great potential for subject-specific learning.
This dual significance of argumentation was effectively summarized by Andriessen et al. (2003) in the book *Arguing to Learn* through the following two expressions: *learning to argue* and *arguing to learn*. In summary, while *learning to argue* involves acquiring the skills that characterize argumentation as a communicative act, such as justifying one's own position or countering another's opinion, *arguing to learn* instead presupposes the use of argumentation in an educational context to achieve a specific learning objective. Although distinct from one another, it is important to emphasize that the two processes are strongly interdependent. In fact, as noted by Schwarz (2009), when arguing to learn, it will be necessary to rely on argumentative skills already possessed by students; at the same time, the learning content of an argumentative discussion may concern argumentation itself, such as valid claims and arguments regarding a certain topic.
Argumentation, understood in both interpretations highlighted by Andriessen, has taken on growing importance in education in recent decades, becoming the subject of research and interest by numerous researchers.
Concerning *learning to argue*, argumentation is conceived as a cross-curricular competence, acquired gradually during the learning process and subsequently applicable to various contexts. Argumentation represents a key competence for citizens of a democratic state, as emphatically stated by Resnick and Schantz in the preface to the book *Dialogue, Argumentation, and Education* (Schwarz & Baker, 2016):

> «[A citizen's] Participation in a democracy depends on the ability to enter fully into the public debates and discussions of the day. This means being able to form a position based on evidence, counter a claim, persuade someone to take another view of an idea, or convince another of the worth of a plan. This means knowing how to keep a conversation going when the parties deeply disagree. These are the skills that allow individuals to shape their own destinies. The same skills will be needed for an educated citizenry to reshape society».
>
> (Schwarz & Baker, 2016, p. xii)

The role of this competence is further elaborated in the preface to the text Argue with me by Kuhn et al. (2017), which reiterates that argumentation represents

> «the primary means citizens in a democracy have for addressing the many matters that affect their common welfare. The premise underlying a democracy is that its citizens possess the skill and disposition to employ these means, to address both smaller and larger issues».
>
> (Kuhn et al., 2017, p. 150)

Regarding *arguing to learn*, research in this area has explored the potential of argumentation for learning specific content across various academic disciplines, including mathematics. In particular, research on this topic has shown that when argumentation is directed at another person, the arguer is prompted to clarify the contradictions and shortcomings of their claims in order to convince their interlocutor, thereby deepening their understanding of the concept in question (Kuhn, 1992).
In this regard, in the introduction to the book *Argumentation and Education*, Muller Mirza and Perret-Clermont (2009) highlight how argumentation is a linguistic, logical, dialogic, and psychological process that has the potential to improve students' understanding, stimulating learning in many ways.
According to the authors, argumentative practices, by requiring students to articulate and justify their own positions, enable «explorative, critical and enquiring approaches to reality: encouraged to test the validity of each other's ideas, the learners are led to formulate objections and counter-objections and to understand a multiplicity of positions» (Muller Mirza & Perret-Clermont, 2009, p. 1).
Research on this topic also recognizes that argumentation has the potential to support the organization of knowledge (Means & Voss, 1996), as well as to facilitate learning by reducing the cognitive load involved, thanks to the dialectical nature of argumentation and peer collaboration. The social dimension of argumentation, in fact, allows for a shared understanding of concepts, with positive effects on students' volitional and motivational aspects.
The didactical potential of argumentative practice has also been recognized in the fields of science and mathematics. Various studies in the field of education highlight how widespread the belief is among students of all ages that mathematics is a static discipline, consisting of a finite system of rules, facts, and formulas (Geisler & Rolka, 2021), in which the validation of statements is often based on the textbook's or teacher's authority (Inglis & Mejia-Ramos, 2009). This belief is often shared by mathematics teachers as well (Handal, 2003). In this sense, through the practice of argumentation, it is possible to convey a vision of science, particularly mathematics, as a lively debate, as is the case with all disciplines. A vision of science in which the exchange of opinions, the coexistence of different perspectives, and the expression of criticism and doubts represent fundamental and legitimate practices for the attainment of new knowledge and solutions to the tasks posed.
From this brief overview of the potential and didactical implications of argumentation, also within a mathematical context, it becomes clear how important it is to focus specifically on two aspects in schools: *learning to argue* and *arguing to learn*. Aware of the strong interdependence between the two, as mentioned earlier, this article will focus particularly on this second conception of argumentation, *arguing to learn*, to also develop the first.

2.1 Argumentation in institutional educational documents

In recent years, the interest in argumentation on the part of educational research has had a significant impact on policymakers and educators who have drafted reference frameworks, international surveys, and curricula for various school levels.
The development of communicative competence, and argumentative competence in particular, is now seen as a crucial, cross-curricular, and transdisciplinary learning goal to be pursued throughout the entire educational journey, starting from the earliest years of schooling.
Several international organizations have issued documents reaffirming this approach.
In the document *Preparing 21st century students for a global society* (National Education Association, 2014), communication and argumentation play a primary role among the fundamental competences to be developed in the education of students so that they can develop critical and communicative skills, which are central for success in school and in an increasingly complex world.
Furthermore, in the *European guidelines on key competences for lifelong learning* (European Commission: Directorate-General for Education, Youth, Sport and Culture [DG EAC], 2019), argumentation is cited as a component of both *literacy competence* and *citizenship competence*. *Literacy competence* includes the ability to «formulate and express one's oral and written argumentations in

a convincing way appropriate to the context. It encompasses critical thinking and ability to assess and work with information» (DG EAC, 2019, p. 6), while *citizenship competence* includes the ability to think critically, as well as the «skills to develop arguments and constructive participation in community activities» (DG EAC, 2019, p. 12).

The argumentation presented in the international documents cited above, described as an essential cross-cutting competence for citizens of the contemporary globalized world, is also identified as an indispensable component of scientific literacy in general, and of mathematical literacy in particular.

The most recent *PISA framework*, specifically dedicated to the assessment of mathematics, dates to 2022. In this document, the skill to present argumentations truthfully and persuasively is described as «a skill that is becoming increasingly important in today's world» (Organisation for Economic Co-operation and Development [OECD], 2023, p. 15). It describes mathematics as a science that offers the opportunity to analyse and transform well-defined objects and concepts through activities such as «reflecting on mathematical arguments and explaining and justifying mathematical results» (OECD, 2023, p. 21) and in which «clarity of contexts and strong emphasis on logical reasoning and rigour at the appropriate level, is a perfect opportunity to practice and develop the ability for this kind of argumentation» (OECD, 2019, p. 41).

Argumentation also appears as a cross-curricular competence to be promoted in national curricula for all school levels, usually associated with two other terms of equal importance in mathematics education: communication and proof. In particular, the topic is central in the Swiss context: *argomentare e giustificare (arguing and justifying)* represents one of the two competence aspects declared at the national level in the document *Competenze fondamentali per la matematica. Standard nazionali di formazione* (Conferenza svizzera dei direttori cantonali della pubblica educazione [CDPE], 2011a) and adopted based on *L'accordo intercantonale sull'armonizzazione della scuola obbligatoria* (CDPE, 2011b) in the Canton of Ticino as well. In the *Piano di studio della scuola dell'obbligo ticinese* (Dipartimento dell'educazione, della cultura e dello sport [DECS], 2022), for example, the ability to communicate and argue for one's own statements is cited both as a cross-curricular competence, included under reflective and critical thinking, and as a cognitive process to be developed specifically within the field of mathematics, which is recognized as having the potential to foster the ability to listen to, understand, and value argumentations and viewpoints different from one's own, and then integrate them constructively with one's own.

The contribution of mathematics in this regard is also recognized by the *Indicazioni nazionali per il curricolo della scuola dell'infanzia e del primo ciclo d'istruzione* (Ministero dell'Istruzione e del Merito [MIM], 2025), which state that: «from a perspective of cultural growth, starting in primary school, the study of science, together and in integration with mathematics, is indispensable for fostering the development of logical reasoning and argumentation skills, critical thinking, language proficiency, and mastery of the Italian language», and in particular «to form informed citizens capable of making informed decisions on issues of global significance» (MIM, 2025, p. 130, translation by the authors).

Argumentative competence is thus described by international and national guidelines as crucial for the formation of citizens in a modern democratic society and, at the same time, of fundamental importance for the development of scientific competence (particularly in mathematics) and a vision of science consistent with its epistemological status.

Argumentation thus appears as a complex competence with numerous potentials, which cannot simply be equated with the ability to reason or prove, as is often understood in the teaching practice of mathematics classrooms, but which necessarily involves consideration of the dialogic context in which it is embedded, the interlocutors to whom it is addressed, as well as the purpose for which it is formulated, as envisaged by the pragma-dialectical theory of argumentation, first theorized by van Eemeren and Grootendorst (1992).

3 The pragma-dialectical theory of argumentation

The pragma-dialectical theory of argumentation is based on the pragmatic, social, and dialectical aspects of argumentation, which is conceived as a critical dialogue aimed at resolving a difference of opinion, in which the participants in the dialogue can be held accountable for their statements.
In particular, in the text *Argumentation theory: A pragma-dialectical perspective*, van Eemeren (2018) defines argumentation as

> «a communicative and interactional act complex aimed at resolving a difference of opinion with the addressee by putting forward a constellation of propositions for which the arguer can be held accountable in order to make the standpoint at issue acceptable to a rational judge who judges reasonably».
>
> (van Eemeren, 2018, p. 3)

This definition of argumentation aligns with the conception of argumentation cited in institutional educational documents, which view argumentation as a tool for citizens capable of supporting a thesis while simultaneously adopting a critical stance toward others' arguments, to collaboratively solve problems, resolve conflicts, and make decisions. The focus on the dialogic dimension of the pragma-dialectical theory of argumentation also aligns this approach with various theories, widely accepted in the educational field, that recognize language and dialogue as fundamental tools for learning and cognitive development (Bruner, 1990; Mercer, 2002; Mercer & Littleton, 2007).
The importance of adopting a pragmatic perspective on argumentation stems not only from its proximity to what occurs in everyday contexts, in democratic life, and in educational settings, but is also supported theoretically, as this perspective allows us to highlight aspects of argumentative discourse that would otherwise be excluded.
As evidence of this, let us consider the case of typical pragmatic fallacies that cannot be described by adopting a logical view of argumentation. The "shifting the burden of proof" fallacy is a clear example of this. This fallacy occurs when the person who has stated a claim asks the interlocutor to prove that the claim is false, rather than providing him/herself argumentations to support it (for a more detailed description, see, for example, van Eemeren et al., 2014). This example clearly shows how the description of argumentation must necessarily include the aspects of conversation management considered by pragmatic perspectives on argumentation.
From a pragma-dialectical perspective, argumentation occurs in communication between two or more interlocutors whenever there is any difference of opinion, whether real or merely imagined by the interlocutors. Such a difference of opinion does not need necessarily to take the form of an actual conflict or dispute; indeed, as highlighted by van Eemeren (2018):

> «more often than not the difference of opinion does not take the shape of a full disagreement involving two opposed standpoints, but remains basic. In that case, the one party has an opinion about something, and the other party does not yet share this opinion but is in doubt as to whether to accept it».
>
> (van Eemeren, 2018, p. 1)

This assumption implies that «when it is presumed that the addressee is not yet convinced of the acceptability of the standpoint at issue, otherwise doing [arguing] so would be pointless» (van Eemeren et al., 2014, p. 2).
From an educational perspective, it is clear that argumentation, as understood here, differs from the

descriptive or explanatory activities typical of school practices; in fact, while the latter aim to clarify a certain idea or statement without the subject matter being challenged through dialogue, on the other hand, argumentation arises with the intent to increase or decrease the degree of acceptance of a certain thesis through the use of arguments for or against the idea in question (van Eemeren & Grootendorst, 2003; van Eemeren & Henkemans, 2016; Walton, 2005).
The definition of argumentation formulated by van Eemeren (2018), which introduces this section, encompasses a series of general characteristics attributed to argumentation, some of which are independent of how this word is used in ordinary language.
In this regard, the author highlights the following four characteristics of argumentation, which are crucial for defining and theoretically addressing it: an argumentation consists of a functional combination of a set of communicative acts; it is, in principle, part of a dialogue; it is an activity of reason; and it appeals to the reasonableness of a rational interlocutor. These four characteristics then give rise to the five analytical elements for interpreting mathematical tasks.

1. An argumentation necessarily consists of the *functional combination of a set of communicative acts* (which may potentially be nonverbal or visual, though they are usually presented in oral or written form) that give the argumentation its structure. This means that pragma-dialectics does not identify the argumentation with its structure, as a complex of logical inferences, but instead considers the argumentation as a set of communicative acts, focusing primarily on the specific functions that the various linguistic acts perform with respect to the management of a difference of opinion (van Eemeren et al., 2014). The analysis of argumentation as a set of communicative and interactional acts is reflected in pragma-dialectics in a foundational metatheoretical principle known as *functionalization*. This principle consists of treating every speech activity as an act with a purpose, thus focusing on how language is used in argumentative practice to communicate and interact, directing attention to the «specific functions that the various kinds of argumentative moves made by the parties involved in argumentative discourse fulfil in managing their disagreement» (van Eemeren, 2018, p. 21).
2. An argumentation is *dialogic*, at least in principle, in that it aims to elicit a reaction from the interlocutor indicating acceptance of the thesis in question. The dialogue in which the argumentative discourse is embedded may be explicit or remain implicit, as in the case of a reader or a passive audience. The role of communication and interaction between the parties in arguing a particular thesis is thus recognized as fundamental from the theoretical perspective of pragma-dialectics, placing it in sharp contrast to approaches that describe argumentation as the product of an individual thought process aimed at establishing the truth of a statement. The conception of argumentation as an interactive exchange in which the contributions of both parties systematically depend on one another has methodological implications for the treatment of argumentation; this metatheoretical principle is called *socialization*.
3. Argumentation is an *activity of reason*, so the arguer can be held responsible for the constellations of propositions advanced (van Eemeren, 2018). The responsibilities that the arguer must assume depend on the propositions stated and the communicative function they serve in the discourse (van Eemeren et al., 2014). These responsibilities can translate into specific interactional obligations. The arguer who has stated a viewpoint has the interactional obligation to defend it when challenged to do so by the listener. The metatheoretical principle that derives from the fact that one can be held responsible for one's own communicative act is called *externalization*.
4. In an argument, *the listener can be regarded as a reasonable judge*; this implies that the arguer is required to convince the listener while adhering to mutually agreed-upon critical standards of reasonableness. This means that argumentation does not aim to have an audience that automatically accept a certain thesis, as can happen in persuasive practices that unscrupulously exploit the feelings of listeners; on the contrary, the goal is to convince the interlocutors by demonstrating

that the shared criteria of reasonableness have been respected, thereby enabling them to reasonably judge the validity of the argumentation (van Eemeren et al., 2014). This point is linked to the metatheoretical principle of *dialectification*, according to which a critical discussion aimed at resolving a difference of opinion must be subject to standards of reasonableness.

In the pragma-dialectical perspective, the concept of reasonableness is distinguished from that of rationality, and the semantic difference between the two is clarified by van Eemeren as follows: reasonableness consists in «using reason in a way that is appropriate in view of the communicative and interactional situation» (van Eemeren, 2015, p. 577), whereas rationality is conceived more generally as the «use of reasoning» (van Eemeren et al., 2014, p. 6).
Van Eemeren's (2018) definition highlights the close relationship between reasoning and argumentation. For the author, an argumentation involves reasoning, and this implies that the arguers can be held accountable for their argumentations and, at the same time, that the recipient of an argumentation must be considered a rational judge, who judges reasonably, based on mutually shared critical standards of reasonableness.
Reasoning is, therefore, a fundamental and central element of argumentation, although argumentation is not equivalent to the verbalization of a reasoning. To distinguish between the two concepts, however, it is necessary to clarify what reasoning is, drawing on psychological research on the subject.

4 Reasoning

In the field of cognitive psychology, Mercier and Sperber (2011, 2018) describe reasoning as a particular type of inference. An inference generally consists of the «extraction of new information from that already available, whatever the cognitive process at work» (a definition that is consistent with that proposed by other researchers in the field of cognitive psychology, such as Moshman, 1998, p. 952). In reasoning, specifically, this act of extraction is accompanied by awareness of one's own thought, that is, by an understanding of the supportive relationship between one or more reasons and a conclusion. It follows that in reasoning, awareness of the reasons is an indispensable characteristic, unlike in the case of inferences; there will therefore be some inferences that are not reasoning.
Mercier and Sperber (2018) thus define reasoning as «the particular process of pursuing this goal [the generation of new beliefs] by attending to reasons» (p. 53), emphasizing the role of reasons and the awareness of those reasons in forming new beliefs. Mercier and Sperber, therefore, use awareness of thought to distinguish between various inferential processes, placing reasoning at one end of a continuum ranging from entirely unconscious inferences to inferences of which we are partially aware. In this sense, according to the authors, inferences «are not distinguished from one another by properties of the inferential mechanisms involved but by the way the process of inference and its conclusion are or are not metacognized» (Mercier & Sperber, 2018, p. 66).
This results in a description of inferential processes that distinguishes, for example, perceptions, intuitions, and reasoning, based on our awareness of their formulation. In the case of perceptions (or memories), we are not at all aware of making inferences, but rather we have the sensation that «what we perceived was immediately present to us, and as if what we remember was retrieved just as it had been stored» (Mercier & Sperber, 2018, p. 62). For example, when looking out the windows of a train and seeing the train next to ours moving, we think it is departing; in reality, we are making an inference. This inference is so spontaneous and unconscious that we only realize we have made it if it turns out to be incorrect, that is, if we realize that it is our train that has departed.
In the case of reasoning, awareness concerns both the conclusions reached and the reasons that support the conclusion.

Mercier and Sperber (2018) also point out that reasoning is not entirely conscious to the individual who engages in it, as it always contains an intuitive component. As for intuition, it consists of an inference characterized by a certain awareness of the conclusion reached, but not of the reasons that were considered. We have an intuition, for example, when we grasp an implicit meaning during a conversation, or when we sense that our father is in a bad mood from the very first words of a phone call. Reasoning, according to the authors, «reasoning is not an alternative to intuitive inference; reasoning is a use of intuitive inferences about reasons» (Mercier & Sperber, 2018, p. 133). In other words, in the case of a reasoning process, the intuitive component lies in the understanding that the reasons considered support a certain conclusion (the authors call this conclusion *reflective* and describe it as *embedded* in an intuitive argument, emphasizing the intuitive dimension underlying any reasoning).
To clarify this point, let us consider, as an example, the case in which a prize is promised if, when choosing between two bags, a white marble is drawn. The first bag contains three white marbles and three black ones, while the second contains four white and three black ones.
One might infer in an intuitive way that, since the probability of drawing a white marble from the second bag is higher, it is preferable to choose the second bag, and this leads to the reflective conclusion of choosing the second bag. The conclusion to choose the second bag is therefore an indirect product of the intuition that a higher probability is a good reason to prefer one bag over the other.
It should also be noted that an argument can be derived through reflection, allowing for the construction of progressively more complex reasoning in which multiple levels of argumentation are embedded within one another. Returning to the case of the two bags, if, for example, our interlocutor did not find the argumentation presented convincing, we could incorporate it into a more general line of reasoning. We could then construct an argumentation of this type: given the same number of black marbles, the number of white marbles contained in the second bag is higher; therefore, the probability of drawing a white marble from the second bag is higher. In this case, the fact that the probability of drawing a white marble from the second bag is higher would constitute a reflective argument, and the choice of the second bag would be a reflective conclusion. The reasoning could be made even more complex by further incorporating the argumentation formulated into a more general one.
In conclusion, according to Mercier and Sperber (2011, 2018), human thought consists of unconscious inferential processes, to the extent that circumstances that lead us to pay attention to the reasons supporting our conclusions are rare. According to the authors, the reasons that prompt us to develop reasoning usually concern the questioning of our conclusions by experience or, even more likely and significantly, by our interlocutors. Dialogue with an interlocutor and dialogue with reality thus represent the two main contexts for the use of reasoning.
The search for valid reasons for our conclusions finds its most natural context in argumentation, where the intent to convince and the other people's critical attitude lead us to reason to support our theses and to make explicit and become aware of a certain inferential process.
In fact, an argumentation often develops once an opinion regarding an issue is already present in some form. The arguer will therefore seek, through reasoning, reasons that justify what they claim (Mercier & Sperber, 2018); thereby making explicit the various inferential steps.
Building on the work of Mercier and Sperber, reasoning has thus been characterized as a specific cognitive activity, marked by awareness of and attention to the reasons involved, highlighting its deep connection to argumentation, since the latter represents the primary context in which reasoning is applied.

5 The relationship between reasoning and argumentation

This section examines the relationship between reasoning and argumentation from the perspective of the pragma-dialectical theory of argumentation and the characterization of reasoning provided by Mercier and Sperber, both of which were presented earlier. It explores the relationship between argumentation and the verbalization of reasoning, as the latter represents a particularly common communicative act in school settings.
From van Eemeren's definition of argumentation, cited above, it emerges that this linguistic act does not coincide with reasoning, but rather that argumentation involves reasoning within a conversational context. Mercier and Sperber (2011, 2018) further hypothesize that argumentation is the most natural context for reasoning, since the latter

> «enables communicators to produce arguments to convince addressees who would not accept what they say on trust; it enables addressees to evaluate the soundness of these arguments and to accept valuable information that they would be suspicious of otherwise».
>
> (Mercier & Sperber, 2011, p. 72)

In particular, the relationship between reasoning and argumentation is described here through the following five analytical elements: verbalization, awareness of reasons, dialogue, difference of opinion, and incentive to argue.

Verbalization. The first fundamental element to be observed concerns the formulation of an argumentation and of a reasoning. While a line of reasoning may remain unexpressed, formulated only mentally, an argumentation must, at least in principle, be expressed or semiotized. If reasoning is a mental process of understanding a relationship, even in a mathematical context, then, according to van Eemeren's definition, for it to be considered an argument, such reasoning must be made explicit in a «communicative and interactional act complex» through «a constellation of propositions» (van Eemeren, 2018, p. 3).
From this perspective, the pragma-dialectical theory of argumentation takes verbalized argumentations as its fundamental object of analysis. A participant in a dialogue will be held accountable only for what they have stated and not for what they have thought; conversely, some rhetorical approaches to argumentation focus on the motivations and attitudes underlying the argumentative discourse. This principle is known as *externalization* and consists of «determining the commitments of the parties based on the way in which they have expressed themselves in the discourse» (van Eemeren et al., 2014, p. 526).

Awareness of reasons. Awareness of reasons represents a common aspect between reasoning and argumentation.
As previously noted, adopting the theoretical perspective of Mercier and Sperber (2011, 2018), indeed, reasoning represents a specific cognitive process through which we search reasons to support a particular conclusion. Reasoning thus involves gaining a progressively greater awareness of the reasons underlying the inferential processes that take place, which are usually unconscious. For the two researchers, the argumentative context represents the most natural setting for the application of reasoning; in this context, attention to reasons is stimulated by dialogue with the interlocutor, whose critical attitude prompts the arguer to seek out and provide reasons that may prove effective and appropriate for the thesis presented. In an argumentative context, the persuasive aim and the need to

be understood by one's interlocutor, as well as the latter's objections, thus lead the arguer to become aware of the reasons supporting their conclusions.

Dialogue. A third point of connection between reasoning and argumentation emerges once again from a reflection on van Eemeren's statement and is based on the dialogical dimension of argumentation. Argumentation is «an interactional act complex directed at eliciting a response that indicates acceptance of the standpoint that is defended» (van Eemeren et al., 2014, p. 5). In principle, therefore, argumentation is always part of a dialogue with an addressee. According to pragma-dialectical theory, the social and interactional nature of argumentation is reflected in the nature and distribution of the communicative acts that the parties involved perform in resolving a difference of opinion (van Eemeren, 2018).
Conversely, as mentioned in sec. 4, reasoning, although it finds its most natural context of application in argumentative contexts, is also employed in problem-solving or to answer tasks for which we do not have an intuitive solution. This observation is also reflected in the relationship between the verbalization of reasoning and argumentation. An argumentation is not limited to a monologue intended to convey, in a one-way manner, a line of reasoning developed in isolation; rather, it constitutes a communicative act embedded within a dialogue and does not concern the sender alone but rather addresses and adapts to the interlocutor.
Reasoning is always involved in the construction of an argument. Still, it cannot constitute the argumentation itself, since the latter does not consist of the thoughts of a single individual but is, by its very nature, the product of cooperation among the participants involved (Muller Mirza & Perret-Clermont, 2009).
In an argument, it is in fact the constant feedback from the interlocutor that allows the arguer to calibrate the argumentations so that they are appropriate and focused on the recipient, and in the same way, it is dialogue that allows for the negotiation of the material premises (i.e., the shared premises) and procedural premises (i.e., the rules to be followed) shared between the parties, which serve as fundamental starting points for the argumentation (van Eemeren, 2018).
This point is also reflected in the extreme didactical relevance of dialogue for learning in general, as confirmed by established pedagogical theories that describe child development as the progressive appropriation of tools constructed within an interpersonal context (Vygotsky, 1978).

Difference of opinion. A fourth point to consider when tracing the relationship between reasoning and argumentation concerns the aspect related to the difference of opinion. In this regard, the author emphasizes that «[...] more often than not the difference of opinion does not take the shape of a full disagreement involving two opposed standpoints, but remains basic» (van Eemeren, 2018, p. 1).
In the everyday context of community life and human action, argumentation typically arises when a collective decision must be made or when the validity of a claim must be established in cases where direct verification is not possible. According to Rigotti and Greco Morasso (2009), it can be said that «the proper scope of argumentation is the area of communal life and of human action» (p. 20). In fact, arguing in everyday life more often concerns action rather than knowledge, and actions often do not pertain to the sphere of general principles but operate in the «field of things that are in a certain manner but could also be in another» (p. 19), and can therefore be changed or implemented through human intervention. In these cases, a difference of opinion arises, the rational resolution of which precisely involves the use of argumentation.
Reasoning, on the other hand, has more general applications: we resort to reasoning whenever we seek to solve a problem, weigh the options of a choice, ask ourselves a question etc. This also occurs individually and without generating a clash of theses or arising from the need to settle a difference of opinion. In these cases, reasoning alternates with intuition, seeking, through backward inference,

reasons in favour of intuitive conclusions, and examining and evaluating those conclusions.
This difference in application is also reflected in the comparison between the verbalization of reasoning and argumentation. When a line of reasoning is articulated, where it does not arise from the need to resolve a conflict, it leads to a structure of the communicative act and a selection of reasons that are based on the conviction of the person who formulated the reasoning, rather than on the pragmatic intent of resolving a doubt. An argument, on the other hand, «always arises in response to, or in anticipation of, a difference of opinion, and the lines of justification that are chosen in the argumentation are contrived to realize the purpose of resolving this difference of opinion in the case concerned» (van Eemeren, 2018, p. 21). Not only the necessity of the argument, but also its structure and the criteria it must meet, are directly linked to the doubt or difference of opinion it aims to resolve.

Incentive to argue. A final and important element to consider concerns the presence of an incentive to argue. When faced with a difference of opinion, in fact, the parties involved must have personal motivations to resolve that difference and thus engage in argumentative discourse. In everyday contexts, although this is not an absolute rule, this stems from the fact that each party involved usually wants the disagreement to be resolved in their favour, and this is translated into the intention to convince their interlocutor of the validity of their own position.
In general, when seeking a solution to a particular problem or considering the various options of a choice, reasoning may have a purely cognitive or epistemic purpose, or it may be aimed at evaluating an intuitive conclusion once that conclusion has been refuted by experience. Argumentation, on the other hand, aims to resolve a difference of opinion while simultaneously seeking the effectiveness of one's communicative act and the maintenance of standards of reasonableness. The arguer seeks not only to ensure that their argumentation is considered reasonable but also that it is effective in convincing their interlocutor (van Eemeren, 2018). To this end, the arguers select the most appropriate starting points, the most suitable type of argument, and structure their argumentations based on the characteristics of their interlocutor so that they are as clear and convincing as possible (van Eemeren, 2018).
Here again, the differences between argumentation and communicative acts, such as the verbalization of reasoning, are evident. The latter may, in fact, serve an exclusively informative purpose, for example, by communicating the reasons identified in support of a certain thesis that have proved convincing to oneself, or by describing the procedure adopted to solve a task. This interesting communicative act, which is legitimate and formative for didactical purposes, differs, however, from a true argumentation, in which the desire to convince by appealing to reason is crucially influenced by the interlocutor, focusing on the search for reasons that can convince the other person as well as oneself. Just consider when a student tries to convince the teacher of the soundness of their problem-solving process when a higher or lower numerical grade on a test is at stake.

6 Argumentation in school

In school settings, there are numerous mathematics activities to which reasoning-related learning objectives are assigned. These activities vary from one another, differing, for example, in the type of task they propose, in the context in which the task is presented, or in the linguistic acts required to complete the activity. This section presents examples of tasks[1] from various contexts, such as the

1. For the original text of the tasks, see the Italian version of this article: https://doi.org/10.33683/ddm.26.19.1.

Italian national assessment tests administered by the *Istituto nazionale per la valutazione del sistema educativo di istruzione e di formazione* (INVALSI), the competitions of the *Rally Matematico Transalpino* (RMT), classroom activities, and a workshop offered during a mathematical festival.
Specifically, several argumentative didactical tasks are analysed in light of the analytical elements identified in sec. 5: *verbalization*, *awareness of reasons*, *dialogue*, *difference of opinion*, and *incentive to argue*, to verify which and how many of these peculiar elements of argumentation, from a pragma-dialectical perspective, are potentially stimulated in such activities. The analysis conducted concerns the formulation of the tasks and the potential context in which they are applied, but not the actual dialogue that emerges from the interaction with and among students.
This analysis can therefore be didactically useful during the design phase. In light of the results of this analysis, we will show how some of these activities interpret argumentation as the verbalization of a process, others as the verbalization of reasoning that also involves awareness of the reasons, and how only in a few cases they attempt to adopt a broader perspective that approaches the pragma-dialectical approach, including dialogic aspects, a difference of opinion to be resolved, and an incentive to argue.
Furthermore, during the analysis, we will discuss certain design factors related to the tasks and the didactical contexts in which they are embedded, which could facilitate or hinder the realization of the analytical elements of pragma-dialectical argumentation, and which should be considered during the design and implementation of argumentative activities in the classroom, to develop students' argumentative skills.

6.1 First example: the verbalization of the procedure

This section presents an analysis of several tasks that are typically considered to be argumentative in the school setting, but which, in our view, are limited to the verbalization of a procedure. The linguistic acts involved in these tasks are more closely associated with description than with argumentation; we note that these are distinct acts, with theoretical differences that were explained in sec. 3.
An example of this type of task, presented here, is taken from the Italian INVALSI national assessments in mathematics. It should be noted that the tasks on the INVALSI standardized tests are answered individually by students, who have a maximum of about one hour to complete approximately thirty tasks. Argumentation is a key competence for all grade levels in the Italian school system, as indicated, albeit with varying emphasis, by the *Indicazioni nazionali*, and as such is included among the mathematical competencies assessed by the INVALSI tests.
Specifically, reasoning is listed among the processes measured by the *Quadro di Riferimento delle prove INVALSI di Matematica* (INVALSI, 2018) and is explicitly mentioned in the *Archivio prove INVALSI di Matematica*[2] in process 600, «progressively acquire typical forms of mathematical thinking (conjecturing, reasoning, verifying, defining, generalizing, …)» (translation by the authors), and in process 610, «use typical forms of mathematical reasoning (conjecturing, arguing, verifying, defining, generalizing, proving, …)» (translation by the authors), as well as among the keywords associated with the tasks: «comparison of arguments», «explanation of arguments», and «justification of arguments» (translation by the authors).
It should also be noted that the *Quadro di Riferimento delle prove INVALSI di Matematica* (INVALSI, 2018) specifies that standardized tests are ill-suited to fully assess the attainment of the competence to construct arguments, thus highlighting the limitations of this assessment tool from this perspective.

The task under analysis is shown in Figure 1 and is taken from the Mat-SNV 2018 test,[3] intended for

2. https://www.gestinv.it/RicercaGuidata.aspx.
3. The abbreviation "Mat-SNV 2018" refers to the mathematics test administered in 2018 by INVALSI's *Servizio Nazionale di Valutazione*.

fifth-grade students. This task has been classified in the *INVALSI Mathematics Test Archive* under both processes 600 and 610 and with the keyword «explanation of arguments».

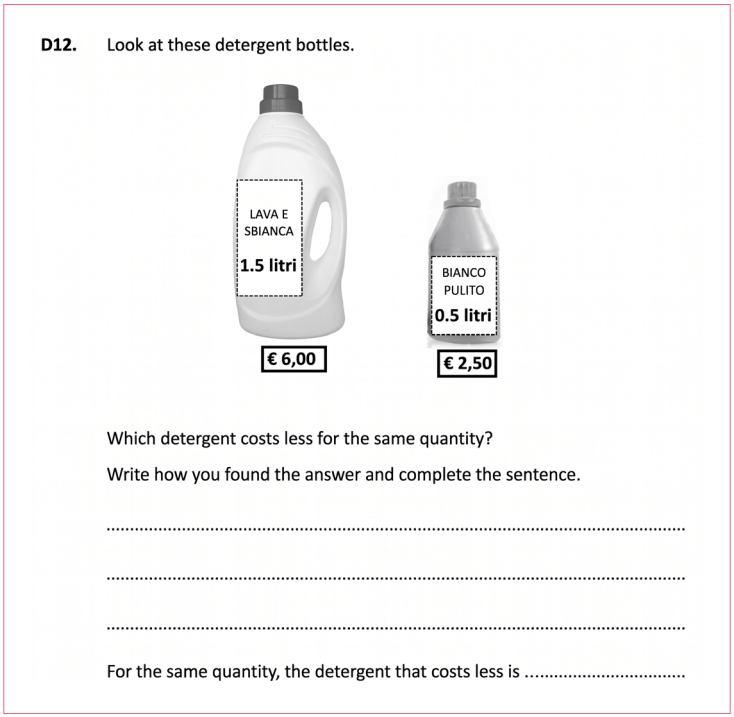


Figure 1. Task 12 from the Mat-SNV 2018 test (translation by the authors).

The task requires the formulation of an answer to a question and the production of a short written text prompted by the request: «Write how you found the answer…», thus addressing the *verbalization* of a solving procedure followed.

As previously noted, the verbalization of a procedure, while serving a formative purpose, does not necessarily imply a search through reasoning because support the chosen procedure and validates the solution reached. In this case, in fact, the mere request to describe the procedure followed does not directly prompt the search for and awareness of the reasons that support it.

The type of task, and above all the specific prompt, therefore, play a fundamental role in students' awareness of the reasons supporting a procedure or a solution.

The cognitive process of reasoning, as explained in sec. 4 through the theory of Mercier and Sperber (2018), is activated in cases where unconscious cognitive processes such as intuition or memory prove to be insufficient, and usually following an encounter with reality or with another person. Consequently, awareness of the reasons involved in an inference is not spontaneously attained by the individual unless it is necessary or explicitly requested. In the example in question, the student might solve the task through intuition or memory. For instance, they might intuit that the cost of the second bottle could be tripled and then compare this value to the cost of the first, or they might recall this procedure from memory if they had already learned it previously. In both cases, the procedure applied could be obtained without the student being *aware of the reasons* that make it valid. According to Mercier and Sperber (2018) and van Eemeren and Grootendorst (2003), awareness of the reasons supporting a certain thesis is a central aspect of argumentation: the argumentative context requires the arguers to clarify their inferential process, identifying reasons that may be convincing to their interlocutor. Furthermore, the wording of the task and the assessment requirements of standardized

tests do not encourage *dialogue*, since the communicative act required to the students is not directed at an interlocutor whose reasons are known (the test evaluator) and whose counterargument can be anticipated, thereby preventing the person advancing the thesis from adjusting their argumentation accordingly.

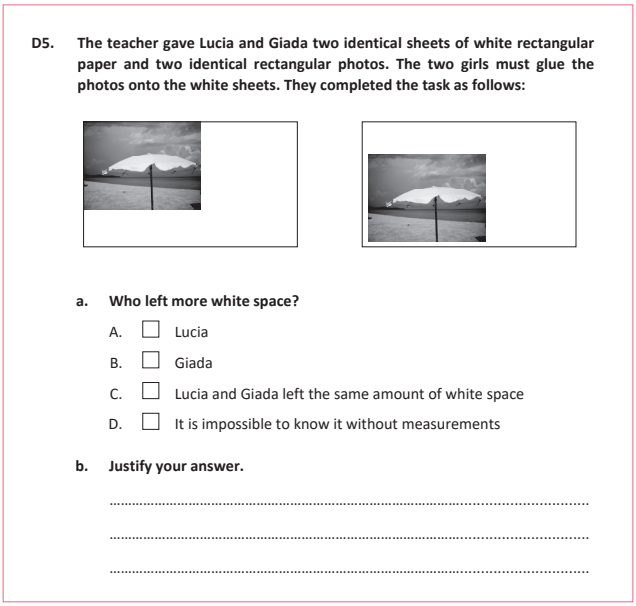
The task does not directly encourage the emergence of a *difference of opinion*, since it does not present competing arguments and reasons among which the students must take a position, by defending their own position while also considering those of others. Nor the task presents a sufficiently broad scenario to encourage the generation of various solutions from which to choose, thus preventing argumentation from being the sole rational tool for managing a difference of opinion.

Furthermore, there is no *incentive* to formulate *an argument,* since the interlocutor to whom the student is addressing, namely, the INVALSI evaluator, does not need to be persuaded of the validity of the proposed solution, as they know the correct answer and the validation criteria. It must be clarified, however, that the requirement to produce an individually written text hinders the emergence of a dialogue, but does not represent an absolute constraint. In general, it is indeed possible to produce an argumentative text regarding a debate topic and within an implicit dialogue, by imagining and anticipating the possible counterarguments of an interlocutor whose prior convictions are known, to convince them of one's own thesis. In the specific case of the INVALSI tests, however, students cannot imagine an interlocutor with these characteristics.

Finally, it should be emphasized that INVALSI's decision to classify this type of task as "argumentative" may have negative implications for teaching practice, as it may reinforce the idea among teachers that argumentation can be reduced to the description of a problem-solving procedure and that argumentative competence in mathematics can be developed solely by presenting this type of task.

6.2 Second example: awareness of reasoning

Another example of a task taken from the INVALSI tests, specifically from the Mat-SNV 2012 test, is the following (Figure 2).



D5. **The teacher gave Lucia and Giada two identical sheets of white rectangular paper and two identical rectangular photos. The two girls must glue the photos onto the white sheets. They completed the task as follows:**

a. **Who left more white space?**

A. ☐ Lucia

B. ☐ Giada

C. ☐ Lucia and Giada left the same amount of white space

D. ☐ It is impossible to know it without measurements

b. **Justify your answer.**

.....

.....

.....

Figure 2. Task 5 of the Mat-SNV 2012 test (translation by the authors).

This is a task intended for fifth-grade students which, like the previous one, has been labelled with processes 600 and 610. This task requires the student to justify the answer. As indicated in the dic-

tionary *Il Nuovo Vocabolario di Base della lingua italiana* by De Mauro (2016), the linguistic act of «justifying» takes on the meaning here of «giving valid reasons for one's choices» (translation by the authors).

In this case, the request to justify the solution should encourage the student to identify and verbally articulate the reasons supporting their choice. The task thus incorporates the first theoretical element of pragma-dialectical argumentation, namely *verbalization*. Specifically, in this instance, the communicative act required to the student is aimed at making explicit the reasons supporting the chosen answer. The second of the analytical elements defined in sec. 5, *awareness of the reasons*, is therefore also present. The correct answer to the task, «Lucia and Giada left the same amount of white space», could in fact be reached at intuitively, without being conscious of the reasons why this is true. However, the explicit request to justify the given answer encourages the use of reasoning to seek out those reasons. According to Mercier and Sperber (2018), reasoning is a cognitive act in which awareness concerns both the conclusions reached, and the reasons involved in support of the conclusion. In this task, reasoning might lead students to argue that "subtracting equal areas from equal areas" is a good (conscious) reason to infer that the resulting areas are equal (in Mercier and Sperber's terminology, this conclusion is called reflective).

We stress again that awareness of the reasons involved in an inference is a fundamental aspect of constructing an argument; from a didactical perspective, if one wishes to develop argumentative competence, it is therefore important to encourage this by explicitly asking students to justify, give reason for, or directly argue for their choices. In this regard, we note that, according to Mercier and Sperber (2018), human thought consists of unconscious inferential processes. As previously mentioned, reasoning is employed when a certain conclusion, reached unconsciously, proves to be incorrect when tested against reality, or when it proves to be ineffective in front of an interlocutor during an argumentative discussion. From a didactical perspective, this consideration leads us to reflect on the fact that the explicit and direct request to identify reasons (through the request to justify or give reason for something) should be accompanied by didactical activities set within argumentative contexts where it is necessary to resolve a difference of opinion, or where intuitive conclusions prove to be incorrect.

Furthermore, as in the previous example (sec. 6.1), the wording of the task and the context of standardized tests do not involve *dialogue*, not even implicitly. The communicative act required to the student is, in fact, directed at the standardized test evaluator, with whom it is not possible to establish shared premises on which to base the argument. Furthermore, the student does not know the evaluator's expectations, nor what reasons might be considered appropriate; this prevents the student from structuring their communicative act based on the interlocutor, as would occur within an implicit dialogue.

In this case as well, as in the previous one, the task neither introduces nor encourages a *difference of opinion*, since the solution to the task can be verified directly and no competing theses are presented for debate.

Linked to this is the lack of an *incentive to argue*: in this case, the student is aware that the interlocutor possesses the correct thesis and mathematically sound reasoning; consequently, the persuasive intent of the communication is lost, and it is driven solely by the (possible) desire to earn a good grade.

The communicative contexts associated with the two tasks presented in this section and the previous one refer to argumentation as the verbalization of procedures or reasoning. This interpretation is quite widespread in school practice and is also reflected in the literature on the topic in mathematics education, as evidenced, for example, by the definition provided under the entry *Argumentation in Mathematics Education* in the *Encyclopedia of Mathematics Education* (Sriraman & Umland, 2020),

in which the argumentations produced by students and teachers during mathematics lessons are defined as

> «a line of reasoning that intends to show or explain why a mathematical result is true. The mathematical result might be a general statement about some class of mathematical objects, or it might simply be the solution to a mathematical problem that has been posed. Taken in this sense, a mathematical argumentation might be a formal or informal proof, an explanation of how a student or teacher came to make a particular conjecture, how a student or teacher reasoned through a problem to arrive at a solution, or simply a sequence of computations that led to a numerical result».
>
> (Sriraman & Umland, 2020, p. 63)

This definition is particularly broad and, at the same time, lacks any reference to the dialogic dimension of argumentation.

6.3 Third example: a potentially favourable context

The third example we consider combines further elements of pragma-dialectical theory, although, as we shall see, only some of these are directly encouraged.

This task is taken from the *Rally Matematico Transalpino* (RMT), a mathematics competition organized from 2001 to 2024 by the *Associazione Rally Matematico Transalpino* [ARMT] (2020), in which the entire class participated. The competition was open to students from elementary school through the first two years of high school. Participants in the competition came from Belgium, France, Italy, Luxembourg, and Switzerland, forming an international group for collaboration and experimentation with didactical materials. The structure of the competition required each class to solve a list of math problems completely on their own, without any support from the teacher. The students therefore had to organize their work internally, dividing up the problems, which were then solved in small groups or individually. Within each subgroup, a discussion might take place to arrive at a shared solution.

ARMT identifies argumentation as a central component of RMT problems, evident both in written explanations and in group discussions during the problem-solving process (ARMT, 2009). In the context of RMT competitions, in fact, students engage in dialogue with one another in an effort to understand the problem's context, during the identification of a solution strategy, and during the final phase, in which they are asked to present their solution in written form, demonstrating how they proceeded to arrive at the solution. Note, therefore, how much the context of the competition itself facilitates the act of argumentation, even more so than the task itself.

RMT problems are also widely used in practice outside the context of the competition, becoming, as the association itself hopes:

> «an integral ("constitutive") part of the mathematics curriculum and its objectives, particularly about fostering a scientific mindset and reasoning: the development of independent learning, the organization of research, rigorous notation, and the ability to argue and communicate results».
>
> (ARMT, 2020, translation by the authors)

The following task is taken from the second test of 2019 and is intended for fourth-, fifth- and sixth-grade students (**Figure 3**).

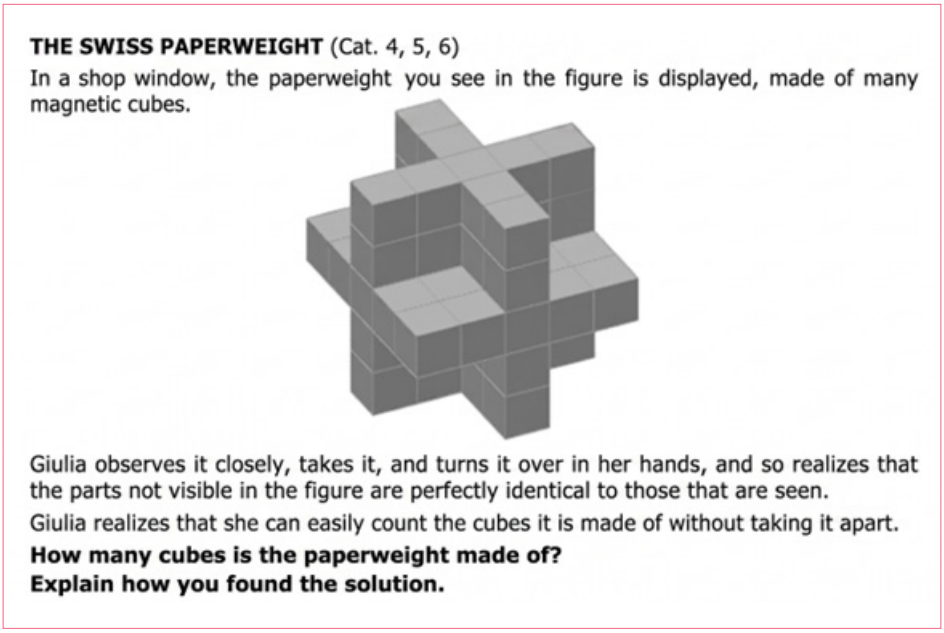


Figure 3. Task 6 from the second test of the RMT 2019 (translation by the authors).

The task shows an image of a paperweight composed of a certain number of cubes. Solving the problem requires visualizing the shape of the paperweight to be able to count the cubes it is made of, and then explaining how the solution has been found. Specifically, regarding this last requirement, the test evaluators refer to the problem-solving procedure used during the solution, as confirmed by the following scoring rubric for written answers provided to the evaluators:

> «4 points: correct answer (61 cubes) with a detailed explanation of the strategy used and any calculations (if it is explicitly stated that 1 to 5 cubes are missing and not visible, a consistent answer is accepted: 56 to 60 cubes); 3 points: correct answer with a description of the procedure followed that is not entirely clear or detailed; 2 points: correct answer without an explanation, or an incorrect answer due to a calculation error, but with a detailed explanation; 1 point: start of correct reasoning (for example, identifying the number of cubes present in certain parts of the figure…); 0 points: failure to understand the problem».
>
> (ARMT, 2006, translation by the authors)

Proceeding with the analysis of the task in light of the analytical elements of pragma-dialectics, the explicit request to explain how one arrived at one's solution presupposes a *verbalization* of the procedure carried out during the resolution; in this case, the meaning attributed to the linguistic act of explaining is consistent with that provided in sec. 3.
As for the students' *awareness of the reasons* involved, this may emerge during the discussion phase within each group, through the argumentations they use to convince their peers, even though the task does not explicitly require justifications for the proposed solution.
Dialogue is encouraged by the structure of the RMT tests, which are solved in groups. Problems are divided among the class, so it is possible for a small group of students to work on solving the task by engaging in dialogue. In this case, the dialogue is explicit; constant feedback from peers compels group members to refine their argumentations to reach a shared decision.
The task does not directly encourage a *difference of opinion*, although one may emerge if two or more students disagree on the procedure to follow during the group work. There is also no difference of opinion with the evaluator, who already possesses the correct solution and valid problem-solving

strategies. A difference of opinion could instead be encouraged by presenting students with various problem-solving strategies to choose from, or by proposing a problem for which it is not possible to establish a single solution, so that the students themselves formulate different strategies and approaches to the problem.

As for the *incentive to argue*, during the discussion phase among students, it may manifest itself through the persuasive intent to propose one's own solution strategy, which is considered correct; however, this persuasive intent is absent when dealing with the external evaluator, as students may assume that the evaluator knows the correct solution and the strategies to reach it.

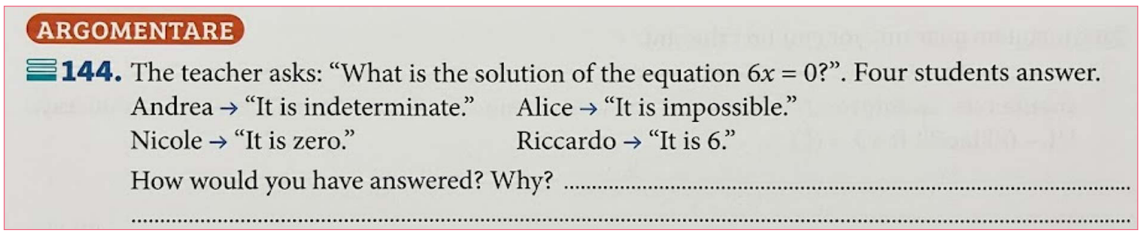
As noted, this task focuses explicitly and exclusively on the verbalization of a problem-solving procedure; the remaining elements, however, while they may emerge through the instruction for students to solve the various problems in the RMT test as a class, are not directly emphasized.

Although the task is solved in a group setting and within a dialogic context during the competition, the verbalization of a specific procedure, as outlined in the criteria provided to the test evaluators, is explicitly required. Therefore, the decision to label this type of task as "argumentative" could once again reinforce teachers' belief that these elements are enough for the development of argumentation in the classroom and the building of students' argumentative competence.

6.4 Fourth example: the presence of a dialogue

Tasks that simulate a dialogue are usually classified as argumentative in textbooks. These tasks often present opposing theses, asking students to choose one and, if necessary, justify their choice. The student is thus called upon to take a position and convince a hypothetical interlocutor to change their mind.

An example of this type of task, taken from the lower secondary school textbook *Tutto chiaro!* (Montemurro, 2021), is as follows (**Figure 4**).


ARGOMENTARE

144. The teacher asks: "What is the solution of the equation $6x = 0$?". Four students answer.

Andrea → "It is indeterminate." Alice → "It is impossible."

Nicole → "It is zero." Riccardo → "It is 6."

How would you have answered? Why? .....

.....

Figure 4. A task taken from the textbook *Tutto chiaro!* (Montemurro, 2021, p. 278, translation by the authors).

Examples of this type can also be found in the INVALSI tests; below is a task intended for fifth-grade students (**Figure 5**). The task is taken from the Mat- SNV 2022 test and is assigned the keyword «justification of arguments» and the dimension «arguing».

D24. In the following dispenser, there are 80 colored balls: 40 are red, 20 are green, and 20 are yellow.
Alice and Marco are discussing what the percentage of red balls inside the dispenser is.

Alice says: "50% of the balls in the dispenser are red."
Marco says: "40% of the balls in the dispenser are red."
Who is right?
Choose one of the two answers and complete the sentence by explaining why they are right.

☐ **Alice is right because** .....

.....

.....

☐ **Marco is right beacuse** .....

.....

.....

Figure 5. Task 24 from the Mat-SNV 2022 test (translation by the authors).

Analysing this task (Figure 5) through the analytical elements of pragma-dialectical argumentation reveals that the task requires verbalization, which is explicitly requested. Specifically, students are asked to explain why the chosen character is right. As indicated in De Mauro's dictionary *Il Nuovo Vocabolario di Base della lingua italiana* (2016), the request to «explain» here means to make explicit «the reason why» (translation by the authors) the chosen answer is correct. Attention to the reasons is also encouraged by the phrase «Alice/Marco is right because», which the student is asked to include as the opening of their written response. This task thus directly encourages *awareness of the reasons* involved, which represents the second of the analytical elements taken into consideration. The presence of a dialogue, albeit simulated, allows for the comparison of opposing theses and arguments. The argumentation could in fact be directed toward the character who advanced the thesis not chosen by the student, thereby qualifying as an imaginary interlocutor with whom to argue. Hypothesizing the opposing party's reasons allows one to argue by organizing one's communicative act in a way that is centred on the interlocutor, providing appropriate reasons and implicitly highlighting the weaknesses of the other party's arguments. In this way, one's argumentation becomes part of an implicit dialogue, in which it is possible to imagine the reasons or counterarguments of one's interlocutor. It must be acknowledged, however, that the presence of multiple options to choose from and the requirement to justify that choice do not guarantee that the activity constitutes an authentic context for argumentation.

The absence of an explicit dialogue, in fact, prevents one from actively negotiating the substantive and procedural premises with one's interlocutor and from engaging in the dialectical process of reaching a mutual agreement, which lies at the heart of argumentation.

As for the need to resolve a *difference of opinion*, the task presents two distinct theses, between which the student is asked to choose. A difference of opinion thus arises, within which the student's communicative act takes place. Nevertheless, it is not possible for the student to formulate their own

thesis, and the situation described does not appear to be a realistic one in which a decision must be made and for which it is not possible to directly verify the solution. In this case, in fact, it is possible to apply a computational algorithm to unambiguously verify the solution, whereas argumentation is typically used in contexts where multiple valid choices are possible and where the best possible choice cannot be determined absolutely. The proposed difference of opinion is therefore artificial; it does not arise in a context where one must choose between two equally valid theses to reach a common opinion.
Finally, although there is an implicit interlocutor to be convinced, the absence of a personal motivation for which it is meaningful to support one of the proposed theses within a realistic context, for example, one in which it is necessary to resolve a difference of opinion to make a decision, hinders the emergence of an *incentive to argue*, and in particular of a persuasive intent.

6.5 Fifth example: resolving a difference of opinion

Regarding the design of didactical contexts in which argumentation serves as a means of resolving a doubt or a difference of opinion, some didactical experiments have sought to create tasks for which there is no clear criterion or procedure for determining a single solution. As an example, we present one of the task taken from Antonini (2018), which was presented to seventh- and eighth-grade students. The researcher drew on game theory to develop problems with which students could identify and whose solutions were uncertain, placing students in a position to formulate hypotheses and argue to reach a shared solution.
While working on the problem, the students involved collectively tried to reach a common solution to the following task:

> «Three musicians, Ada, Bea, and Ciro, are contacted to play at a party. They can perform alone, in pairs, or as a trio. The fees set by the event organizer are as follows: 100 euros to Ada (if she plays alone), 150 euros to Bea (alone), 180 euros to Ciro (alone), and 600 euros to the trio if they play together. If they play in pairs, the earnings will be as follows: 400 euros to the pair Ada and Bea, 300 euros to Ada and Ciro, 420 euros to Bea and Ciro. Putting yourselves in the shoes of the three musicians, try to discuss the offer and explain how Ada, Bea, and Ciro might agree. Remember to adequately give reasons for your statements».
>
> (Antonini, 2018, p. 37, translation by the authors)

Students are asked to give reasons for their statements: *verbalization* is thus directly encouraged.
As for the *awareness of reasons*, as indicated by De Mauro's dictionary *Il Nuovo Vocabolario di Base della lingua italiana* (2016), the linguistic act of «giving reasons for [something]» takes on the meaning here of «explaining with appropriate motivation or justification» (translation by the authors). Furthermore, the task presents a situation like everyday contexts where reasoning is applied, contexts in which the need to convince an interlocutor leads us to seek reasons that may prove effective. During the discussion phase, students must therefore necessarily pay attention to the reasons supporting their own solution and those of others to reach a common answer.
The task is solved collectively: this places the argumentations within an explicit *dialogue* in which it is possible to critically evaluate others' arguments, allowing the participants to establish the substantive and procedural premises essential to the argument.
About the *difference of opinion* within which the argumentation takes place, we note that the task presented has no definitive solution; this implies that, during the problem-solving process, students are encouraged to formulate their own solutions and to resolve the resulting conflict through reasoned dialogue. In this case, argumentation is in fact the only tool for resolving differences of opinion and determining which decision to make. In this sense, the context of the task resembles real-world situations where argumentation finds its most natural setting. In such situations there is often no

clear criterion for reaching or verifying the validity of a particular solution to a task, making argumentation an indispensable tool for decision-making.
Finally, regarding an adequate *incentive to argue*, and particularly the persuasive commitment on the part of the arguer, we observe that this will be felt more strongly the more the arguers perceive the thesis they are defending as important, as happens in the context of everyday life and human action. In the example in question, the resolution of the task has no direct consequences for the arguer; it is not a matter of deciding or arguing in favour of a thesis that is dear to the one who supports it. The proposed situation instead requires an effort of empathy on the part of the students in a scenario that may not be familiar to them; therefore, there may not be a sufficient incentive to argue.

6.6 Sixth example: incentive to argue

As we showed in sec. 5, pragma-dialectical argumentation requires an appropriate incentive to take place, given that in everyday contexts, the intent to persuade is the most common motivation for producing an argument.
In this regard, here is a significant example that occurred during the fourth edition of the festival *Matematicando. A spasso con la matematica per le strade di Locarno* which took place in 2024 in the city of Locarno (Switzerland).
This event is a biennial mathematics festival in which, in addition to mathematics-themed performances, sections and classes from various school levels conduct mathematics workshops aimed at peers and citizens. Within this framework, argumentation serves as the means to convince one's audience, both young people and adults, of the validity of mathematical concepts, a prerequisite for the success of the activity.
In particular, the example presented here stems from the workshop *Probability* (Pezzi & Cosci, 2026), designed for primary and lower secondary school students, which was organized and run by classes at those same grade levels. During the school year, in preparation for the festival held in May, each class engaged in activities related to probability, and subsequently, the students selected and designed together four stations to be presented in a workshop format during the event. The tasks presented at the stations were chosen to elicit possible erroneous intuitive interpretations in the field of probability, as identified in the relevant literature.
During the festival, the workshop was run by the students, who welcomed the various participating classes and divided them into four groups. Each group visited the four stations in a round, like in a game show, and had a certain number of coins to use throughout the entire course. During the festival, the students running the stations were responsible for managing the activities, presenting the tasks at each station, tracking bets and winnings, and finally explaining the solutions to the participating students.
Here below is a description of one of the four stations.
The student organizers showed their peers a wheel with numbers from 1 to 10, explained how it worked, and invited the contestants to try it. They then proposed the following task:

After spinning the wheel three times, which of these number sequences is most likely to happen?

> After spinning the wheel three times, which of these number sequences is <u>most likely</u> to happen?
> ☐ 10 – 10 – 10 ☐ 1 – 2 – 3
> ☐ 2 – 6 – 9 ☐ None of the above; they all have the same probability.
>
> (Pezzi & Cosci, 2026, p. 4, translation by the authors)

This task relates to the "misconception of representativeness" (Fischbein & Schnarch, 1997), which involves estimating the probability of an event based on how representative that event is of the class of events to which it belongs. For example, if we consider two lottery draws: "1, 2, 3, 4, 5, 6" and "39, 1, 17, 33, 8, 27", the second sequence of numbers, consisting of unordered numbers, appears more representative of a random draw than the first. Based on this consideration, one might conclude that the first sequence has a lower probability of being drawn, whereas both have the same probability of being drawn. For a deeper exploration of the tasks addressed in the workshop, see the didactical worksheet *Probability* (Pezzi & Cosci, 2026).
To choose the correct answer, the contestants had the option to spin the wheel and discuss together which option to choose and how many coins to bet. After sharing these choices, the students had to communicate them to the organizers. At this point, the student organizers invited their peers to explain their answer and, if it was incorrect, had to try to convince them that their choice was wrong (**Figure 6**).

Figure 6. A photograph taken during the workshop.

The didactical activity in the task thus involves verbalization by both the student organizers and the participants.
An *awareness of the reasons* behind decisions could emerge at various points during the activity: the different stages of discussion among participating students, as well as between the student organizers and the participants, involved reaching a shared conclusion and persuading one's counterparts so that the game could continue. As stated in **sec. 5**, such situations represent the most natural context for applying reasoning, since it becomes necessary to seek out and articulate appropriate reasons if a conclusion reached intuitively, as is to be expected in this circumstance, is not accepted by the interlocutor. Awareness of the reasons involved was thus indirectly encouraged during the activity, even though the production of reasons was not explicitly required.
In this context, *dialogue* is explicitly encouraged. Throughout the entire activity, in fact, and in all phases of discussion, the communicative acts of the students involved were directed at interlocutors whom they sought to persuade and who could express their doubts and counterarguments regarding the theses presented. This condition makes it possible to structure argumentations based on direct feedback from interlocutors, thereby realizing the social and interactional nature of argumentative practice.
The *difference of opinion* among the discussion participants was encouraged by the choice of the task. The selected task was, in fact, phrased in such a way as to elicit responses linked to the misconception of representativeness. The intuition that leads to the conclusion that an ordered sequence is a very special case of a random draw is particularly widespread; this aspect could generate differences

of opinion between the student organizers and the participants, but also among the participants themselves. Although the problem can be solved by applying probability theory, if the students are familiar with it, the presence of the representativeness misconception and the fact that probability had been covered only briefly in the classes participating in the workshop allowed for genuine differences of opinion to emerge during the activity.
Finally, there was a clear *incentive to argue* one's case, that is, a desire to persuade among the students involved in the didactical activity. In this example, the context played a fundamental role: both the student organizers and participants were personally engaged thanks to the festival setting and the dynamics of the prize game. All of this helped create an environment in which students felt motivated to argue their points, both during the discussion phase among participants, where the dynamics of the prize game encouraged them to persuade their peers, and during the final explanation phase. In this last phase, in fact, the organizing students' persuasive intent was further supported by the context and their own roles. After working extensively on the tasks in the preceding months, and having to justify to the participants why they had won or lost their coins, the organizers resorted to numerous argumentations to convince their counterparts, for example by displaying a table of all verifiable cases, allowing the contestants to spin the wheel multiple times, or highlighting analogies with other games the participants had already solved.

7 Conclusions

The work illustrated in this article, based on the pragma-dialectical theory of argumentation (van Eemeren, 2018) with the supplement of the theoretical description of reasoning given by Mercier and Sperber (2018), has made it possible to identify and characterize the following five analytical elements: verbalization, awareness of reasons, dialogue, difference of opinion, and incentive to argue. The choice of this theoretical perspective allows for the treatment of argumentative competence in a manner that is consistent with the expectations of national and international educational guidelines, the reference frameworks of international assessment programs, and the curricula of various countries. This approach integrates the social, pragmatic, and dialogic dimensions of argumentative practice, to foster both the process of arguing to learn and the ability of learning to argue; the latter is considered a fundamental competence for the future citizens of a democratic society.
The analysis conducted validated the effectiveness of the five identified analytical elements, which provide a useful framework for determining whether and to what extent diverse didactical tasks meet the characteristics of pragma-dialectical argumentation.
In particular, the analysis revealed that some well-known tasks examined in this article, which are typically regarded as designed to develop students' argumentative skills, incorporate only a few of the analytical elements under consideration, focusing primarily on the verbalization of a problem-solving process or reasoning, without situating the resulting communicative act within a social and pragmatic context that involves dialogue, difference of opinion, or the incentive to persuade. This type of interpretation of argumentation as the verbalization of a procedure or reasoning represents a valid didactical goal from a mathematical perspective, especially in preparation for learning mathematical proofs; however, in our opinion, such activities are not sufficiently effective in developing students' argumentative skills in line with the key competences required to future active and critical citizens of a democratic society. Therefore, the choice to label this type of task as "argumentative" in textbooks, standardized tests, or various circulating resources can have negative repercussions on teaching practice, as it may reinforce the idea that argumentative competence can develop without considering further pragmatic elements that are fundamental to argumentation.

The analysis of these tasks, conducted using the five analytical elements, also clarified the role of certain aspects of the task and its context, to stimulate pragmatic-dialectical argumentation in students. One aspect that emerged concerns the *type of task* proposed to encourage argumentation, which often involves a single correct answer intended to test students' knowledge. Yet, as we have highlighted from the literature (Rigotti & Greco Morasso, 2009), a rich and open-ended task would be more suitable for fostering it, since it would more closely resemble everyday and community situations in which a decision must be made, without it being possible to clearly and directly verify the correctness of one choice over another. In this way, the task could prompt the generation of diverse opinions, as it cannot be solved by applying a known algorithmic procedure, thereby allowing the fundamental function of argumentation to take place: the rational management of a difference of opinion in an attempt to reach a shared belief (van Eemeren et al., 2014).

Another aspect that has emerged concerns the *linguistic act* required to students in mathematical tasks, which influences what is expected from them. The type of act (to describe, explain, justify, give reason for something etc.) significantly shapes the student's response and should therefore be specified with care. In this regard, we have noted that different choices in this sense, such as requests to describe a procedure or reasoning, to give reasons for something, or to justify, have implications for verbalization and awareness of the reasons involved, encouraging students, more or less explicitly, to focus on the reasons underlying the various situations presented.

Another aspect concerns the presence of an *interlocutor*, whether imaginary or real. For the student's communicative act to be directed toward an imaginary interlocutor, it is possible, for example, to describe a simulated dialogue in the task, providing valid and sufficient reasons for all the arguments presented. In this way, the student will be able to formulate an argumentation directed at and tailored to an interlocutor whose position and reasons they know, and whose counterarguments they can anticipate. In this way, the argumentation will be embedded in an implicit dialogue, in which the arguers do not react directly to objections raised in real time, but deliver a monologue in which they anticipate the possible doubts or counterarguments of a specific interlocutor or audience. An explicit dialogue, on the other hand, is possible when the interlocutor is present, is not already convinced by the thesis being presented, or does not already know the answer to the task and is willing to engage in dialogue with the arguer to arrive at a shared position. Indeed, the theoretical framework has highlighted how dialogue is fundamental for negotiating the premises of one's own arguments, while simultaneously allowing one to understand others' reasons and adjust one's own argumentations accordingly, thereby realizing the social and interactional nature of argumentation (van Eemeren, 2018).

Finally, one last important aspect concerns the *social context* of the activity, in which the student can formulate their own thesis while feeling motivated to defend it and personally engaged. This social dimension is a decisive factor in incentivizing the student to argue. It has, in fact, been shown that an incentive to argue, such as a persuasive intent, is a fundamental ingredient for the establishment of an argumentative context. The propensity to convince using reason, and even more so the willingness to be persuaded when faced with good arguments, are in fact two fundamental elements of argumentative competence.

In conclusion, it can be said that the analytical elements highlighted and the design aspects that emerged from the analysis make it possible to structure didactical contexts to foster the development of an act of argumentation which is consistent with the pragma-dialectical perspective. The perspective proposed can be taken to promote the development of those reflection, communication, and argumentation skills called for by national and international reference frameworks for the growth of informed and critical citizens who actively participates in a democratic society.

Bibliography

Andriessen, J., Baker, M., & Suthers, D. (Eds.) (2003). *Arguing to learn: Confronting cognitions in computer-supported collaborative learning environments.* Springer. https://doi.org/10.1007/978-94-017-0781-7

Antonini, S. (2018). Giocare con la matematica: Argomentare, modellizzare e costruire significati. *Quaderni di ricerca in didattica*, *2*, 13–18. https://flore.unifi.it/handle/2158/1244500

Association Rallye Mathématique Transalpin. (2006). Il fermacarte svizzero. In *Banca di problemi del RMT.* ARMT. https://www.projet-ermitage.org/ARMT/navi_fic2.php?code=3d75-it&flag=1&langue=it&enonce=27rmtii_it-6&w=0

Association Rallye Mathématique Transalpin. (2009). Collezione di giornalini. In *Banca di problemi del RMT.* ARMT. https://www.projet-ermitage.org/ARMT/navi_fic2.php?code=nu36-it&flag=1&langue=it&w=

Association Rallye Mathématique Transalpin. (2020). *Concezione RMT.* ARMT. Retrieved April 1, 2025, from https://armtint.eu/concezione-rmt/

Bruner, J. (1990). *Acts of meaning: Four lectures on mind and culture.* Harvard University Press.

Conferenza svizzera dei direttori cantonali della pubblica educazione. (2011a). *Competenze fondamentali per la matematica.* CDPE. https://edudoc.ch/record/96785/files/grundkomp_math_i.pdf

Conferenza svizzera dei direttori cantonali della pubblica educazione. (2011b). *L'accordo intercantonale sull'armonizzazione della scuola obbligatoria (concordato HarmoS) del 14 giugno 2007.* CDPE. https://edudoc.ch/record/100376/files/Harmos-konkordat_i.pdf

Dipartimento dell'educazione, della cultura e dello sport. (2022). *Piano di studio della scuola dell'obbligo ticinese.* DECS. https://www.pianodistudio.ch

De Mauro, T. (2016). *Il Nuovo Vocabolario di Base della lingua italiana.* Internazionale. https://dizionario.internazionale.it

European Commission: Directorate-General for Education, Youth, Sport and Culture. (2019). *Key competences for lifelong learning.* Publications Office of the European Union. https://data.europa.eu/doi/10.2766/569540

Fischbein, E., & Schnarch, D. (1997). *Brief report*: The evolution with age of probabilistic, intuitively based misconceptions. *Journal for Research in Mathematics Education*, *28*(1), 96–105. https://doi.org/10.5951/jresematheduc.28.1.0096

Geisler, S., & Rolka, K. (2021). "That wasn't the math I wanted to do!" – Students' beliefs during the transition from school to university mathematics. *International Journal of Science and Mathematics Education*, *19*(3), 599–618. https://doi.org/10.1007/s10763-020-10072-y

Handal, B. (2003). Teachers' mathematical beliefs: A review. *The Mathematics Educator*, *13*(2), 47–54. https://doi.org/10.63301/tme.v13i2.1863

Inglis, M., & Mejia-Ramos, J. P. (2009). The effect of authority on the persuasiveness of mathematical arguments. *Cognition and Instruction*, *27*(1), 25–50. https://doi.org/10.1080/07370000802584513

Istituto nazionale per la valutazione del sistema educativo di istruzione e di formazione. (2018). *Quadro di riferimento delle prove INVALSI di matematica*. INVALSI. https://invalsi-areaprove.cineca.it/docs/file/QdR_MATEMATICA.pdf

Kuhn, D. (1992). Thinking as argument. *Harvard Educational Review*, *62*(2), 155–179. https://doi.org/10.17763/haer.62.2.9r424r0113t670l1

Kuhn, D., Hemberger, L., & Khait, V. (2017). *Argue with me: Argument as a path to developing students' thinking and writing* (2nd ed.). Routledge. https://doi.org/10.4324/9781315692722

Means, M. L., & Voss, J. F. (1996). Who reasons well? Two studies of informal reasoning among children of different grade, ability, and knowledge levels. *Cognition and Instruction*, *14*(2), 139–178. https://doi.org/10.1207/s1532690xci1402_1

Mercer, N. (2002). *Words and minds: How we use language to think together.* Routledge. https://doi.org/10.4324/9780203464984

Mercer, N., & Littleton, K. (2007). *Dialogue and the development of children's thinking.* Routledge. https://doi.org/10.4324/9780203946657

Mercier, H., & Sperber, D. (2011). Why do humans reason? Arguments for an argumentative theory. *Behavioral and Brain Sciences*, *34*(2), 57–74. https://doi.org/10.1017/S0140525X10000968

Mercier, H., & Sperber, D. (2018). *The enigma of reason.* Harvard University Press. https://doi.org/10.4159/9780674977860

Ministero dell'Istruzione e del Merito. (2025). *Indicazioni nazionali per il curricolo della scuola dell'infanzia e del primo ciclo d'istruzione.* https://www.mim.gov.it/documents/20182/10554370/curricolo_web.pdf/f91c31a0-5ed4-65f3-bfea-fb49adaba55f?version=1.0&t=1773224873548

Montemurro, A. (2021). *Tutto chiaro! Aritmetica-Geometria. Con Quaderno E Prontuario. Ediz. Tematica.* DeAgostini Scuola.

Moshman, D. (1998). Cognitive development beyond childhood. In D. Kuhn & R. Siegler (Eds.), *Handbook of child psychology: Vol. 2. Cognition, perception, and language* (5th ed., pp. 947–978). Wiley.

Muller Mirza, N., & Perret-Clermont, A.-N. (Eds.) (2009). *Argumentation and education: Theoretical foundations and practices.* Springer. https://doi.org/10.1007/978-0-387-98125-3

National Education Association. (2014). *Preparing 21st century students for a global society: An educator's guide to the "Four Cs".* https://thinkcreatesharegrow.weebly.com/uploads/1/7/8/0/17807859/a-guide-to-four-cs.pdf

Organisation for Economic Co-operation and Development. (2023). *PISA 2025 science framework*. OECD. https://pisa-framework.oecd.org/science-2025/

Pezzi, D., & Cosci, L. (2026). Probability. In Centro competenze didattica della matematica (Eds.), *Schede didattiche.* Matematicando, SUPSI. https://www.matematicando.supsi.ch/risorse-didattiche/probability/

Rigotti, E., & Greco Morasso, S. (2009). Argumentation as an object of interest and as a social and cultural resource. In N. Muller Mirza & A.-N. Perret-Clermont (Eds.), *Argumentation and education: Theoretical foundations and practices* (pp. 9–66). Springer. https://doi.org/10.1007/978-0-387-98125-3_2

Schwarz, B. B. (2009). Argumentation and learning. In N. Muller Mirza & A.-N. Perret-Clermont (Eds.), *Argumentation and education: Theoretical foundations and practices* (pp. 91–126). Springer. https://doi.org/10.1007/978-0-387-98125-3_4

Schwarz, B. B., & Baker, M. J. (2016). *Dialogue, argumentation and education: History, theory and practice.* Cambridge University Press. https://doi.org/10.1017/9781316493960

Sriraman, B., & Umland, K. (2020). Argumentation in mathematics education. In S. Lerman (Ed.), *Encyclopedia of mathematics education* (pp. 63–66). Springer International Publishing. https://doi.org/10.1007/978-3-030-15789-0_11

van Eemeren, F. H. (2015). *Reasonableness and effectiveness in argumentative discourse: Fifty contributions to the development of pragma-dialectics* (Vol. 27). Springer International Publishing. https://doi.org/10.1007/978-3-319-20955-5

van Eemeren, F. H. (2018). *Argumentation theory: A pragma-dialectical perspective* (Vol. 33). Springer International Publishing. https://doi.org/10.1007/978-3-319-95381-6

van Eemeren, F. H., Garssen, B., Krabbe, E. C. W., Snoeck Henkemans, A. F., Verheij, B., & Wagemans, J. H. M. (Eds.) (2014). *Handbook of argumentation theory.* Springer Netherlands. https://doi.org/10.1007/978-90-481-9473-5_1

van Eemeren, F. H., & Grootendorst, R. (1992). *Argumentation, Communication and Fallacies. A Pragma-dialectical Perspective.* Lawrence Erlbaum.

van Eemeren, F. H., & Grootendorst, R. (2003). *A systematic theory of argumentation: The pragma-dialectical approach.* Cambridge University Press. https://doi.org/10.1017/CBO9780511616389

van Eemeren, F. H., & Henkemans, A. F. S. (2016). *Argumentation: Analysis and evaluation* (2nd ed.). Routledge. https://doi.org/10.4324/9781315401140

Vygotsky, L. S. (1978). *Mind in society: Development of higher psychological processes.* Harvard University Press. https://doi.org/10.2307/j.ctvjf9vz4

Walton, D. (2005). *Fundamentals of critical argumentation.* Cambridge University Press. https://doi.org/10.1017/CBO9780511807039