Characterizing prospective teachers' knowledge in/for interpreting students' solutions[1]

Dare senso alle risposte degli studenti: la conoscenza interpretativa nella formazione insegnanti

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Abstract / In this work, referring to the framework of the Mathematical Knowledge for Teaching (MKT) we will talk about interpretative knowledge, that is the particular knowledge involved in the processes of teacher interpretation. In order to investigate this particular knowledge and clarify its characteristics and dimensions, we have designed a questionnaire to which we asked a group of future teachers to interpret some productions given by the pupils to a problem concerning rational numbers. Using the MKT construct, we have found that, in the interpretation and evaluation by future teachers of the answers productions by pupils, an important role is given by the domains of the Common and Specialized Knowledge of Contents (CCK and SCK).

Keywords: prospective teachers training; interpretative knowledge; noticing; rational number; representations operational composition.

Sunto / In questo lavoro, facendo riferimento al quadro della Conoscenza Matematica per l'Insegnamento (Mathematical Knowledge for Teaching, MKT) parleremo di conoscenza interpretativa, ossia quella particolare conoscenza coinvolta nei processi di interpretazione degli insegnanti. Al fine di indagare e chiarirne le caratteristiche e dimensioni, abbiamo costruito un'attività in cui si richiede di interpretare alcune risposte date dagli alunni ad un problema riguardante i numeri razionali. Abbiamo proposto questa attività ad un gruppo di maestri in formazione e, utilizzando il costrutto MKT, abbiamo rilevato che, nell'interpretazione delle risposte degli alunni, un ruolo importante è giocato dai domini della Conoscenza Comune e Specializzata dei Contenuti (CCK e SCK).

Parole chiave: formazione dei futuri insegnanti; conoscenza interpretativa; osservazione; numeri razionali; rappresentazioni; composizione operazionale.

1 Introduction

Nowadays, an important aspect of teaching is promoting students' reflection upon the effectiveness of their own (and others') reasoning and representations chosen to solve a problem. This task requires, among others, the teachers' ability to make sense of and provide feedback to students' solution processes and to support them in their developing knowledge of mathematics. It is also widely acknowledged how this skill

differs from those associated with a traditional view of the teaching of mathematics, conceived as merely a set of definitions and rules. For example, Ball (1993) argues that in the "reform" of mathematics teaching, students/children should be viewed as unpredictable thinkers. Making sense of and supporting mathematical thinking of such unpredictable thinkers requires a precious and flexible mathematical sensitivity on the part of the teachers.

These peculiar features of mathematical knowledge needed for teaching are well grasped and framed by the Mathematical Knowledge for Teaching (MKT) conceptualization (Ball, Thames & Phelps, 2008). Among other things, teachers' knowledge must include those topics that allow them to understand the students' answers – above all when they differ from standard reasoning, i.e. those answers that are more common and frequently used by the teachers themselves. Such understanding of students' answers implies also being able to evaluate if a solution can be considered mathematically valid and/or generalizable to other situations; also working on the given solutions (even the incorrect ones) in order to explore the cognitive processes put in place by the students in a mathematically valid and significant way (for a particular perspective see Borasi, 1994). For this reason, teachers' knowledge should include a broad range of strategies and representations for problem solving.

In this article we report a specific work developed for identifying particular features and dimensions involved in prospective teachers' MKT while interpreting and giving sense to students' productions. With this perspective in mind, we have designed a sequence of tasks focused on the emergence and the discussion of prospective teachers' knowledge involved in this interpretative/evaluative activity. This choice was prompted by the awareness that such work represents one of the core elements in contributing to develop the mathematics' knowledge that the prospective teachers will use in their classroom practice (see for example Rowland, 2008). For these reasons, these tasks can be framed as professional learning tasks, or PLTs, which are activities that simulate one with which the prospective teachers will have to deal with in their professional career (Smith, 2001).

In this article we are going to discuss an activity, structured through a sequence of questions, that requires to give sense to some of the students' answers to a problem involving rational numbers. The way in which the task is conceptualized allows us to explore different and equivalent ways of representing and expressing the same amount (of chocolate) using graphical and numerical representations; they involve both decimal and fractional representations. The choice of such a diversity also allows us to explore the connections between symbols and their semantic referents having recourse to the idea of *operational composition of a number* (Subramaniam & Banerjee, 2011).

In the following sections we will present first the theoretical framework, then the structure of the designed activity and some of the students' productions chosen for interpretation, and in the end some of the results obtained from the analysis of the answers from a group of Italian prospective teachers.

2   Theoretical framework

Teacher knowledge, and in a broader sense, his convictions on the subject and on the dynamics of the process of teaching and learning play a key role in practice. The idea, introduced by Shulman (1986), of several forms of knowledge strongly influenced the research in the teaching of mathematics. Shulman develops a general framework to classify both domains and categories of teachers' knowledge, regardless of the topic. In particular, of the seven categories defined, only three of them are somehow related to the topics to teach: (a) knowledge of content, (b) Pedagogical Content Knowledge (PCK) and (c) knowledge of curriculum. Starting from this first model several articulations and further studies have been developed.

From among the different conceptualizations of teacher knowledge, the Mathematical Knowledge for Teaching (MKT) (Ball et al., 2008, **Figure 1**) effectively captures the particular features of mathematical knowledge needed for teaching.
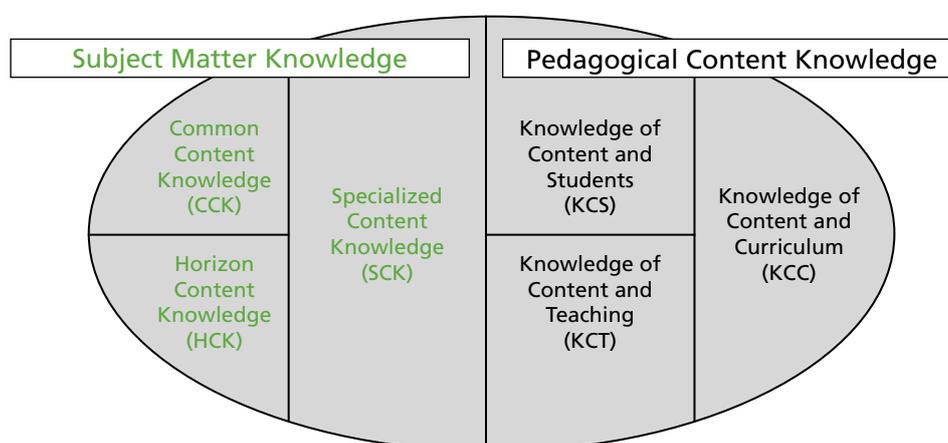

Figure 1
Conceptualizations of Mathematical Knowledge for Teaching (MKT).

The work of Ball and colleagues (Ball, Hill & Bass, 2005; Ball, Thames & Phelps 2008) is particularly relevant in this regard: researchers recognize and define three kinds of Pedagogical Content Knowledge (PCK – right side of the model in **Figure 1**): Knowledge of Content and Students, Knowledge of Content and Teaching, and Knowledge of (Content and) Curriculum. Moreover, MKT introduces a new dimension of knowledge, referring to the specific mathematics topic, the Subject Matter Knowledge (SMK – the left side of the model in **Figure 1**) – complementary and non-reducible to the PCK. Such SMK entails the particular features of mathematical knowledge, referring to a specific mathematical topic, required by the teacher to develop his classroom's practice. Thus he can support the students in developing their comprehension and knowledge of that specific mathematical topic – for example, allowing them to understand what they do and why. Such particular features complement the mathematical knowledge that also other professionals (engineers, economists, programmers, etc.) use in their line of work – such as, for example, the one employed for solving correctly a mathematical problem, and that in this model is referred to as Common Content Knowledge, CCK. CCK embeds aspects of mathematical knowledge which are only needed in the teaching process (for example, identifying and understanding wrong answers, Specialized Content Knowledge – SCK) and regard-

ing the awareness of how the mathematical topics are related with each other (Horizon Content Knowledge).

One of the tasks of teaching is to make sense of students' solutions and help them to develop their mathematical knowledge. The knowledge involved in such a task (we will call it *interpretative knowledge*) owns a peculiar and specific nature that motivates our use of the MKT framework to investigate this particular knowledge belonging to prospective teachers. In particular in our inquiry about *interpretative knowledge* we focus mainly on the Common and Specialized Content Knowledge (CCK and SCK) sub-domains of MKT. Indeed, in evaluating and giving sense to students' solutions of a mathematical task the CCK, used both in teaching and in other settings that uses mathematics related to that specific mathematical topic, is of course fundamental. But at the same time this interpretative work needs also another kind of knowledge. Indeed, besides knowing how to solve a certain problem, it is crucial that teachers have the knowledge that allows them to understand the mathematical reasoning behind calculations, definitions, and problem-solving processes. Moreover, teachers should possess a rich and broad set of examples, strategies and representations for problem solving, that allows them to make sense not only of the kind of solutions they are familiar with, but also to non-standard answers, reasoning and strategies. For this reason we believe that the *interpretative knowledge* is tied to the SCK and intertwined with the noticing ability of the teacher, namely the ability to increase the quantity and at the same time refine the quality of the observed aspects in the students' educative processes (Mason, 2001). This involves teachers' ability to make informed choices in contingency moments (Rowland, Huckstep & Thwaites, 2005) and to respond to situations as they emerge, in ways that supply sustainable mathematical knowledge to students.

Our preference for the MKT conceptualization over others derives also from the nature, focus, and aims of the work we are developing. Indeed, the content of the sub-domains of the MKT is considered as a relevant starting point for designing tasks for the mathematical preparation of teachers In our perspective, by considering the importance of a practice-based approach with the goal of developing the SCK, we assume also as essential that prospective teachers experience the same kind of situations they will encounter both in practice and in their professional life. In this work we present activities that require to interpret pupils' problem-solving processes, which the prospective teachers will have face in their professional life (Smith, 2001). The activity we present, in particular, concerns the teachers' interpretative knowledge in a problem involving division sharing and fractions in primary schools. Fractions are among the most complex mathematical concepts that children encounter in their primary education years (Newstead & Murray, 1998). Moreover, the pervasiveness of fractions in many other specific mathematical domains (e.g., operations, probability, and measurement) makes them a crucial and strategic mathematical topic of inquiry. According to Kieren (1995), students' difficulties with fractions can be traced back to the fact that, on the one hand fractions, as particular representations of rational numbers, present many links with other ways of representing the same entities (such as decimals alignment etc.), on the other fractions have several different meaning (such as distribution, measure, operator, ratio quantities) These complexities, and the well-known difficulties, strongly justify the importance of improving teachers' training about this specific mathematical topic (for another interesting research experience see Campolucci, Maori, Fandiño Pinilla & Sbaragli, 2006).

3 Method

Our sample is composed of prospective primary school teachers taking mathematics education courses at universities in the countries (Portugal, Norway and Italy), where the authors served as lecturers. An open questionnaire with a set of tasks was designed and translated into three languages: Italian, Portuguese and Norwegian. The assignments aimed at accessing (and discussing) prospective teachers' interpretative knowledge, in a problem involving fractions. Although we gathered data in parallel (November 2012), our intention was not to carry out a comparative study, but rather to use the diversity between the contexts as an element contributing to a richer understanding of the research topic. However, in this study, for sake of space, we focus only on data from 108 Italian teachers.

3.1 Task structure

Tasks have, or at least they should have, a primary importance in the teaching-learning process as a core way of constructing and developing learners' abilities, knowledge and competences. Clearly the way such abilities and knowledge are perceived and developed depends of the different kinds of learners (e.g., students, prospective teachers); these differences should influence the nature and the focus of the proposed tasks. In that sense, tasks for teachers are perceived differently from the tasks for their students. Thus, in conceptualizing tasks aimed at developing teachers' knowledge, one must take into account the specificities of such knowledge and, as we said, the framework of the MKT can serve as guidebook in this matter.

Focusing in tasks for teachers' training, our approach considers as a core element a link with aspects of teachers' practice, intertwined with the possibility of allowing teachers to have similar kind of experiences we (as teacher educators) expect they will provide their students with. This means allowing prospective teachers to feel involved in a mathematical activity having a twofold vision–both as teachers and learners. Indeed, through such approaches they can experience similar situations of pleasure, difficulty, and enjoyment as students exploring the mathematics involved without being concerned with the didactical component, but at the same time, with a suitable guide, they can improve their noticing capability (Mason, 2001; Davis, 1997) and develop the mathematical knowledge and competences needful for the work of teaching. In that sense, we consider that tasks for teachers built on the request of solving a mathematical problem by themselves and afterwards giving sense to students' productions to the same problem could be seen both as a rich mathematical activity and as a professional learning task (Smith, 2001).

With this idea we are working on a broader research project focusing on designing tasks aimed at improving teachers' competence and interpretative knowledge. Here we focus on a task about rational numbers. For the conceptualization of this task we focus on the complex relationships amongst the several representations of the same rational number (pictorially, arithmetically, using natural language), but also on different possible ways to express the same quantity by adding different fractions. Indeed we believe that working on several representations of the same rational number is a possible threshold to appreciate some aspects of these mathematical entities. As underlined by Subramaniam and Banerjee (2011):

«the expression reveals how the number or quantity that is represented is built up from other numbers and quantities using the familiar operations on numbers. This interpretation embodies a more explicit reification of operations and has a greater potential to make connections between symbols and their semantic referents. The idea of operational composition of a number, we suggest, is one of the key ideas marking the transition from arithmetic to algebra».

(p. 100)

In that sense, in order to promote (prospective) teachers' knowledge for teaching rational numbers, the proposed tasks should focus on developing their knowledge, allowing them to appreciate the mathematical structure of the topic. In doing so, it is important to make prospective teachers explore different ways of interpreting and representing different solutions of a problem.

The task is grounded in a problem for primary students:

"Teacher Maria wants to explore with her students some notions associated to the concept and the nature of fractions, with this aim she prepares a set of problems including the following one:

What amount of chocolate would six children get if we divide five bars equally among them?

1. Try yourself to solve the problem giving a graphic interpretation of the answer".

In order to allow and also open teachers' reflections, the first question of the task asks the (prospective) teachers to solve the problem for themselves (give a solution to the problem), and only afterwards solvers were asked to give sense to some students' productions to the same problem, with the following requests:

2. For each of the following students' solutions say if you consider it mathematically adequate or not by arguing your idea.

3. For the cases in which you think the student's solutions is not adequate, think of a possible feedback to give to that student.

These requests have a twofold aim: on the one hand to involve the prospective teachers in something that they will meet in practice, which is evaluating and interpreting the students' problem-solving processes and, on the other hand, to work on their beliefs and knowledge both mathematical and pedagogical, by proposing them some appropriately chosen solutions.

3.2 The choice of students' productions to interpret

In this paragraph we present, explain, and discuss the chosen productions that we ask to interpret, by showing the link between the graphical representations they presented and their arithmetical counterpart. The choice of the children' solutions aimed at reflecting on the equivalence of different fractional and decimal representations of a same quantity, on the operations with fractions and on the role of the whole.
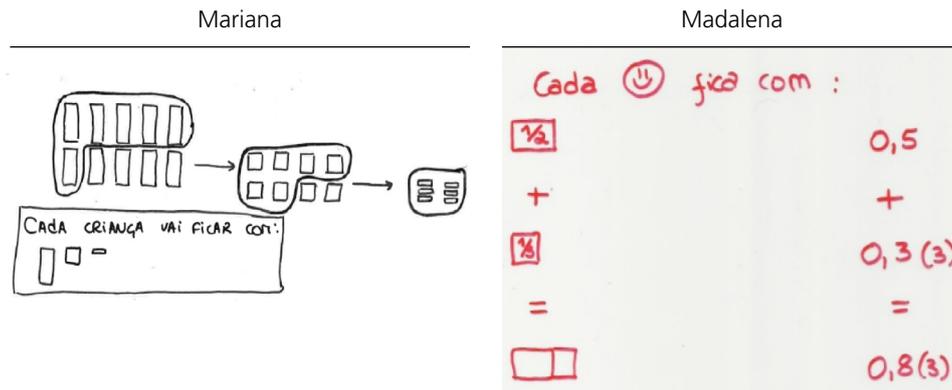
Mariana	Madalena

In Mariana's answer (**Figure 2**), the five chocolate bars were represented by rectangles divided in halves, obtaining ten equal parts (5 is equivalent to $\frac{10}{2}$). The first encircled amount shows the possibility of distributing among the 6 children 6 parts of a bar leaving out after this first distribution four halves ($\frac{4}{2}$). Through an arrow, the four pieces left out of the first distribution are subjected to a new division in halves, obtaining eight equal parts ($\frac{4}{2}$ is equivalent to $\frac{8}{4}$), six of them ($\frac{6}{4}$) being distributed among the 6 children, leaving out, in this second distribution, two of them ($\frac{2}{4}$). Through another arrow, each of the two pieces is divided again in three equal parts, obtaining in this way 6 smaller pieces ($\frac{6}{12}$) which may be distributed among the 6 children. In a sketching box below, are drawn the parts that each child gets. Although the division is correct, the student does not provide any numerical representation of the quantity she obtains, which is given only in terms of drawings, but it could have as a numerical representation ($\frac{1}{2}+\frac{1}{4}+\frac{1}{12}=\frac{5}{6}$).

In the second protocol (**Figure 3**) Madalena represented the corresponding amount of chocolate for each child using different representations. In fact, the amount of chocolate that each child will get at the end of the subdivision is represented both graphically and numerically, writing numbers both in fractional form ($\frac{1}{2}+\frac{1}{3}$) and in decimal form ($0,5+0,\bar{3}$). There are several aspects that lead us to choose this answer as interesting for discussion. In this protocol, besides the difficulties of handling the infiniteness of the decimal representation, such as that of the decimal periodical numbers 0,33 and 0,83 (notice the different representation of the periodical cipher in Portugal). Moreover, in this solution only the answer of the final quantity obtained by each child is present without any reference to the division process, therefore it becomes interesting from an interpretative point of view picturing it.
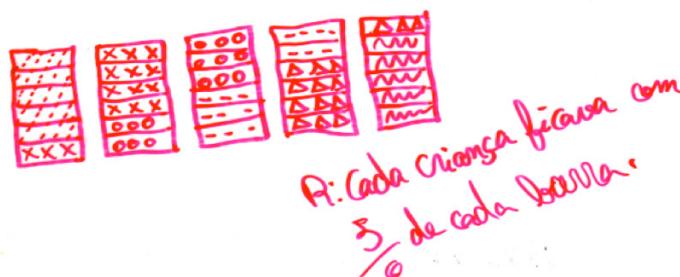
Sofia

In Sofia's production (**Figure 4**) we recognize five rectangles representing the five chocolate bars, divided into six pieces each. Sofia then highlighted 6 groups of 5 pieces each filling them with different patterns. As final answer Sofia writes, "Each child will get $\frac{5}{6}$ of each bar" which, if it were true, would mean that each child would get $\frac{25}{6}$ of a bar (this same mistake occurs also in Inês' production, **Figure 5**). Although the given answer was not correct, the subdivision is coherent from a graphical point of view and allows us to explore meaningfully several non-canonical representations of a same number, and the commutative property: $\frac{5}{6}=\frac{1}{6}+\frac{4}{6}=\frac{2}{6}+\frac{3}{6}=\frac{3}{6}+\frac{2}{6}=\frac{4}{6}+\frac{1}{6}=\frac{5}{6}$, which correspond to the subdivisions outlined by the 6 filling patterns.
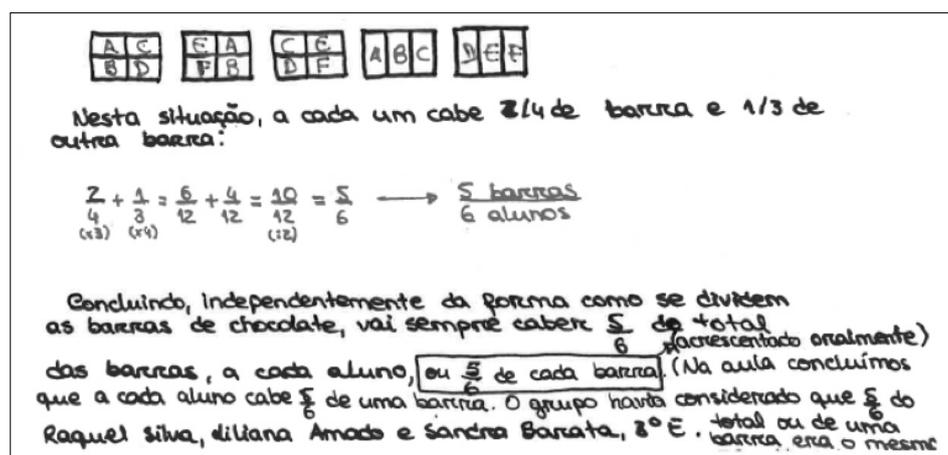

Figure 5
Inês' production.

*"In this situation, each child gets $\frac{2}{4}$ of a bar and $\frac{1}{3}$ of another bar:
$\frac{2}{4}+\frac{1}{3}=\frac{6}{12}+\frac{4}{12}=\frac{10}{12}=\frac{5}{6} \rightarrow \frac{5\,barrette}{6\,alunni}$

▲"In conclusion, independently of how the chocolate bars are divided, each student gets $\frac{5}{6}$ of the total, or $\frac{5}{6}$ of each bar. In class we conclude that each child gets $\frac{5}{6}$ of a bar. The group considered that $\frac{5}{6}$ of the total or of a bar was the same."

Finally the solution of Inês (**Figure 5**) allows prospective teachers a reflection upon several additional issues with respect to those emerging from the previous solutions. The chocolate bars are represented by 5 rectangles, of which the first 3 are divided in 4 parts, the last 2 in 3 parts. Moreover, Inês, unlike Sofia which uses filling patterns, uses the first 6 letters of the alphabet to denote the 6 children to whom distribute the parts of the bar. Although the final numerical and pictorial answer is correct, there are several inconsistencies between what is represented by drawing, what is written in mathematical language, and what is complemented by natural language For the pictorial representation, the expression corresponding to the amount of chocolate of each child would be $\frac{1}{4}+\frac{1}{4}+\frac{1}{3}$. In other words, also in this case the interpretation of this solution involves exploring the operational composition of the number through its different equivalent expressions: $\frac{1}{4}+\frac{1}{4}+\frac{1}{3}=\frac{2}{4}+\frac{1}{3}=\frac{6}{12}+\frac{4}{12}=\frac{10}{12}=\frac{5}{6}$. The arithmetic way of finding the common denominator can be used as a starting point to discuss with the prospective teachers the meaning behind this process for adding fractions. Finally, the solution expressed in natural language results to be incorrect and highlights the complex question of the role of the whole, which is one

of the core points in the difficulties with fractions.

The focus of the presented task is on explicitly developing teachers' mathematical knowledge concerning rational numbers through the request to interpret and evaluate particular children's solutions. The particular choice we made about the children's solutions included in the task we invite (prospective) teachers to explore, in order to reflect on the equivalence of different fractional and decimal expressions of one same amount, the relationship amongst them with pictorial representations, the operations with fractions and the role of the whole.

4 Results and discussion

We start by first presenting and discussing prospective teachers' answers to the problem of dividing the chocolate bars and we will analyze their comments and reflections when interpreting the students' answers.

In the first part of the task, namely when they had to answer individually to the problem, the majority (55 out of 108) of the prospective teachers presented a solution using only natural numbers. These solutions correspond to a graphical subdivision of each bar into six pieces. They concluded that each child would get five pieces In **Figure 6** we reported one of these solutions as an example. This kind of answer appears even more relevant if we consider that the text accompanying the problem explicitly says: "The teacher Maria uses this problem to explore some notions associated to the concept and the nature of fractions with her pupils."
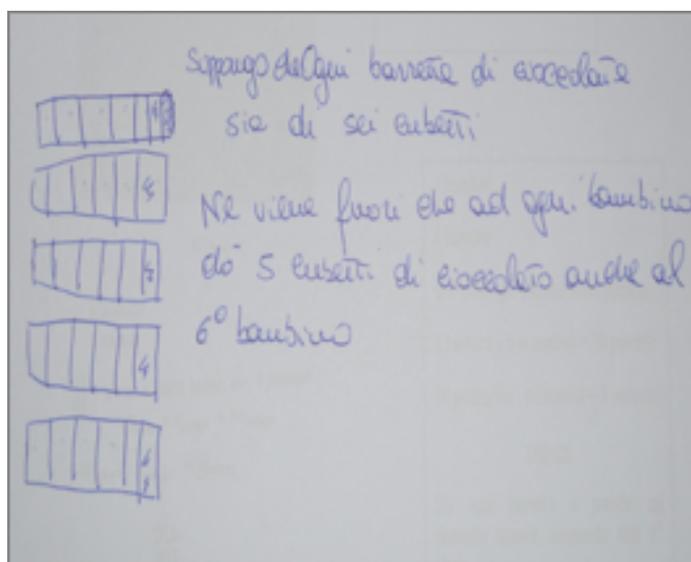
Figure 6
Prospective teachers solution "I suppose that each chocolate bars is composed of six cubes. It comes out that I give 5 chocolate cubes to each child also to the sixth child".

Despite this, more than one half of the prospective teachers chose to remain within the domain of natural numbers, revealing evidence of a lack of familiarity with the mathematical notion of fraction. In other answers, some opted for different uses of fractions in some cases combining the reasoning about the distribution of the pieces with the expression of the pieces in terms of fractions. Among the prospective

teachers who used fractions to answer the problem (46 out of 108), we found some inappropriate use of them. Typical wrong answers in terms of fractions are "each child gets $\frac{5}{6}$ dof each bar" (11 out 46) or "each child gets $\frac{1}{6}$ of one bar" (3 out 46) revealing difficulties in understanding the role of the whole. Similar results were obtained in a previous study (Ribeiro & Jakobsen, 2012).

 We also found answers that used decimals (8 out of 108). The most frequent decimal answer was: 0.83. This first quantitative analysis about teachers' own solutions is useful to frame our particular sample as characterized by a poor CCK about fractions. As confirmation of this we could observe that in the following task about evaluating and interpreting the correctness of student solutions, a really little number of prospective teachers (4 out 108) was able to detect the mathematically incorrectness of the expression "each child gets $\frac{5}{6}$ of each bar".

The main observation concerning the qualitative analysis we carried out on the prospective teachers' comments and making sense of students' solutions to the problem, is that many of them find difficulties in interpreting solutions different from their own, similar to what was expressed by Kuhlemann (2013). Here are some of the comments to the presented solutions: "Sofia's solution appears disorderly, all those signs make the idea confused", "Inês has just complicated her life," "Mariana follows a more intricate path". What is interesting here is that if on the one hand they are not able to detect the mathematical inadequacy of the expression "each child gets $\frac{5}{6}$ of each bar", on the other side they are ready to disapprove their subdivision of the bars that, at a more careful mathematical eye, match with two different sum of fractions of the right amount of chocolate each child gets. Comments as these reveal a big difficulty to leave one's own space of solutions, a space evidently consisting of a single element, making thus impossible to appreciate and understand students' different solution strategies. If in the above comments this negative propensity toward a variety of approaches and reasoning is only visible in transparency, some other prospective teachers explicitly said that a correct answer should necessarily be similar to what they do or think. An example of such feeling is in the following:

> «I consider SOFIA correct [capital letters are in the original text]: to put it better, I mean that they translate my own THOUGHT in the problem solution (…) I consider Marianna's solution mathematically inadequate at the level of understanding, I mean that it is not understandable on a large scale, at first not to myself. A priori I would not consider it wrong, I would skip it now and analyze it in a second moment».

This comment - "I mean that they translate my own THOUGHT" reveals an approach not inclined to interpret students solutions when they are different from our own reasoning paths. Of course this seems a very natural human attitude, and to some extent we could accept it from someone who acts in contexts other than teaching, but we all agree that a teacher, as well as a practitioner, needs to overcome it by developing particular sensitivity and insight – skills which are linked to the SCK. In this sense we believe that prospective teachers need to train in order to be able to interpret children's solutions. On the other hand, one could think that this is the kind of knowledge that will be developed over time with practice. However, our data confirms the research evidence that shows, how this is not completely true:

> «I'm a teacher since ten years and I do think that these solutions are very confusing. I always try to make the visual images as clear as possible and I try to lead my pupils to do the same. In this case the reasoning paths are very messed up and lead to confusion».

In this sense we can say that SCK and the ability to interpret and make sense of student answers do not naturally develop over the course of years of work experience, but are something that requires a special attention from an educational point of view. In question 3 we asked also to imagine possible feedbacks for the students and we found that prospective teachers in case of answers deemed wrong or confused often proposed to show their own solution to pupils, as we can see in the following comment:

> «Mariana's solution is not clear so the first question would be: what does this representation mean? After I have listened her answer, I would show her my own representation».

Moreover it is interesting to notice that among those who revealed a good CCK in the problem regarding the division of chocolate bars (giving for example an answer in terms of fractions), some perceive student solutions, even the inadequate ones, as a possibility and a good starting point to work with the class. In a way some of them, as hoped, see wrong answers as opportunities to develop students' knowledge and awareness on subdivisions and fractions:

> «The first thing I would do is to invite the children to copy the solutions of Sofia, Madalena and Inês onto one poster, and then I would try to make them notice similarities and differences among them. Some of these solutions require much care on the part of the teacher, who has to decode what the pupil meant and help children give sense to different solutions».

These answers reveal very precious and valuable pedagogical insights. We can observe a certain link between these proposals and Mason's idea of *noticing*:

> «(…) to increase sensitivity to notice opportunities to act, while at the same time, to have come to mind in the moment when they are relevant, a range of possible appropriate actions».
>
> (Mason, 2001, p. XI)

5 Conclusive remarks and future perspectives

In this work we presented an activity based on a mathematical problem about rational numbers, designed to investigate and develop the interpretative knowledge of prospective elementary teachers. In the article we presented the students' productions chosen for the prospective teachers' interpretation activity, explaining the rationality and the sense of the choice made. This activity was used in several teacher trainings classes in Brazil, Italy, Norway and Portugal. In this article we discuss some

results obtained implementing the task of dividing the chocolate bars in a group of 108 prospective Italian teachers. The quantitative analysis on prospective teachers' answers to the fraction problem showed evidence of a somehow poor CCK about fractions. Moreover, the designed tasks gave us the opportunity to access also other sub-domains of MKT. In particular, the qualitative analysis, showed that the prospective teachers' highlight difficulties in dropping their own space of solutions and appreciating and understanding different students' solution strategies, as well as a lack of SCK.

One of the possible future research roads is to deepen the links between CCK and SCK (and its content) in teacher's interpretation ability. Finally it should be added that, after administering a questionnaire, we dedicated two hours of collective discussion aiming at developing the prospective teachers' MKT and, at the same time bringing out and discussing their beliefs about mathematics and its teaching-learning processes. Indeed we believe that the proposed task could represent a good example of PLT to explore with prospective teachers, in order to work simultaneously on MKT and their own convictions.

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