The role of text comprehension in the process of mathematizing and modelling

Il ruolo della comprensione del testo nel processo di matematizzazione e modellizzazione

Elena Franchini*, Alice Lemmo° and Silvia Sbaragli*
*Dipartimento Formazione e Apprendimento – SUPSI, Locarno
°Università di Palermo

Abstract / This article analyses the answers given to an item related to the *standardized Mathematics test* administered in May 2015 to all fifth grade primary students in Canton Ticino. Starting from the statistical results emerged from a sample of 508 selected protocols, a qualitative analysis on a smaller sample of 174 students of first year of secondary school has been performed, to trace the causes of the difficulties emerged in the resolution. From this analysis we found that many wrong answers are related to difficulties in understanding the item text, in particular to linguistic interpretation. Considerations made may provide teachers tools for diagnosing specific difficulties and suggesting "intervention areas".

Keyword: reading comprehension; verbal problems; mathematize; modelling; linguistic difficulties.

Sunto / In questo articolo vengono analizzate le risposte fornite a un interessante item relativo alla *Prova standardizzata di matematica* somministrata nel maggio 2015 a tutti gli allievi di quinta elementare del Canton Ticino. A partire dai risultati statistici emersi da un campione di 508 protocolli selezionati, viene presentata un'analisi qualitativa effettuata su un campione più ristretto di 174 studenti di prima media, allo scopo di rintracciare le cause delle difficoltà emerse nella risoluzione. Da questa analisi si rileva come le risposte sbagliate di diversi studenti siano legate a difficoltà nella comprensione del testo dell'item, in particolare a difficoltà di interpretazione linguistica. Le considerazioni effettuate possono fornire strumenti all'insegnante per la diagnosi di specifiche difficoltà e per suggerire "zone d'intervento".

Parole chiave: comprensione del testo; problemi verbali; matematizzare; modellizzare; difficoltà linguistiche.

1 Introduction

In May 2015 the Canton Ticino *standardized Mathematics test* was administered to assess the mathematical content knowledge of 5th grade students in two areas: *Numbers and calculation and Quantities and measures* and three competences: *Mathematizing* and *modelling, Perform and apply* and *Know, recognise and describe*. The project has a two fold objective: to provide monitoring information about the educational system and to give teachers, directors and inspectors indications related to the performances of particular classes (Crescentini, 2016).

At a later time, the *Dipartimento dell'educazione, della cultura e dello sport* asked the *Centro competenze Didattica della matematica of Dipartimento formazione e apprendimento – SUPSI* to conduct a more accurate analysis of the data collected

adopting an interpretative point of view based on mathematics education theories. Such an analysis has the aim to highlight strengths and weaknesses in the students' performances. We chose to deepen the analysis of the *Mathematizing and modelling* process, which represents a key component in the mobilization of mathematical competences. In this paper we present a deep analysis of 5th and 6th grade students responses to a specific item of the standardized Mathematics test. These responses point out difficulties mainly related to the lack of linguistic comprehension of the items' text.
In the research report of the project (Sbaragli & Franchini, 2017) there are other items where we present similar difficulties shown by the students. This kind of analysis could be a useful tool for teachers, allowing them to recognize specific students' difficulties and/or ability and to suggest "intervention areas".

2 Theoretical framework

2.1 The *Mathematizing and modelling* process

The *Mathematizing and modelling* process refers to the activity of organizing and analysing a real situation through mathematical instruments, which means translating, reorganizing, and (re)constructing a problem presented in a real context in the mathematical and symbolic world, and viceversa (Jupri & Drijvers, 2016).

The notion "mathematization" stems from the *Realistic Mathematics Education* (RME) theory, developed in Holland in 1968 starting from the ideas of Freudenthal. He suggested working with students starting from real and not purely mathematical contexts, considering reality a crucial component of teaching and learning mathematics, both as a source and a context in which we can apply mathematical ideas (Freudenthal, 1968; 1991; Treffers, 1987; 1991).

According to RME theory, the term "reality" has a wide connotation: it may be referring to real life, to a fantasy world or to mathematical situations when they are significant and imaginable by students, so that, for example, the essential elements of the proposed situation have been previously experienced and understood by the student (Freudenthal, 1991; Gravemeijer, 1994; Van den Heuvel-Panhuizen, 2000; Van den Heuvel-Panhuizen & Drijvers, 2013). Therefore, in general, we consider as "reality" the natural, social and cultural environment in which the individual lives, as well as its fantasy aspects. As Freudenthal says (1983, p. ix, cited in OECD, 2007), «our mathematical concepts, our structures and ideas have been invented as instruments to organize the phenomena of the physical, social and mental world».

From a didactical point of view, developing the ability to apply mathematics to understand and solve real situation/problems is currently worldwide considered one of the main objectives of mathematical education (Eurydice, 2011; NCTM, 2000; OECD, 2006; 2010; 2013; 2016). As Wheeler says (1982, p. 1), «It is more useful to know how to mathematize than to know a lot of mathematics».
The *Piano di studio della scuola dell'obbligo ticinese* (DECS, 2015) (Curriculum of the Canton Ticino compulsory education) suggests the use of significant learning situa-

tions starting also from contexts outside the school, from daily life experiences. They allow you to work on the ability to use concepts, principles and methods of mathematics in order to understand, explain, examine and represent reality. The objective is to act with awareness on it, in order to manage and use different representations and models, to formalize and generalize the contents they are exposed to and come up with a suitable interpretation of the information they gather.

These skills are part of the process of *Mathematizing and modelling*, i.e. a key competence for the training of the student's mathematical thinking which, as research shows, should be developed from elementary school (Jones, Langrall, Thornton & Nisbet, 2002).

For this reason, in the mathematical frameworks of the main international surveys PISA (OECD), TIMSS and INVALSI national (CDPE, 2011) and regional ones (Sbaragli & Franchini, 2014, 2017) it is emphasized the need to assess the learning of the students on this process, not only to measure the knowledge and skills in the mathematical field, but also the ability to relate these knowledge with the contexts of action that must be addressed.

In OECD (2004) a particular cycle is outlined within the mathematization process, taken up and underlined also in OECD (2013, 2016), which we can summarize in the following aspects:

1. Start with a "real" problem.
2. Organize the problem according to mathematical concepts.
3. Gradually trim away the reality through processes such as making assumptions about which features of the problem are important, generalising and formalising (which promote the mathematical features of the situation and transform the real problem into a mathematical problem that faithfully represents the situation).
4. Solve the mathematical problem.
5. Make sense of the mathematical solution in terms of the real situation.

Using the terminology proposed by PISA this cycle is described through the identification of some processes that are essential for the management of the solution of the problem: to *formulate* the problem, i.e. translate it from the natural language to the formalized language of the discipline, *employ* your own knowledge to give an answer to the identified problem, *interpret* and *evaluate* the relevance of the supposed solution in relation to the original context of reality.

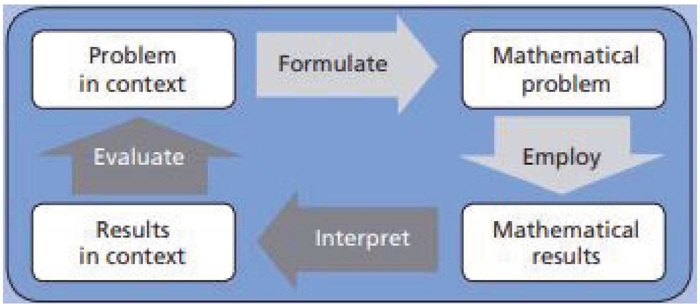


Figure 1
Mathematization cycle from PISA (OECD, 2013).

The ideal cycle presented in Figure 1 refers to a "problem in context". The solver of this kind of problem tries to identify the mathematical aspects relevant for the solution, erasing everything that is irrelevant to the solution, thus finding a structure, an abstract and ideal model of the situation (for example a formula, an algebraic expression or equation, a scheme) based on the hypotheses he has elaborated, on the concepts and relationships he has identified. In this way, the solver transforms the "context problem" in a "mathematical problem", which is a problem that is manageable with mathematical instruments, concepts and procedures.

A model could be defined as «a system of conceptual frameworks used to construct, interpret, and mathematically describe a situation» (Richardson, 2004, p. viii). Modelling provides the identification on the part of the student of the mathematical structure within the problem posed (English & Watters, 2004a).

The first process of the cycle is *formulate*, which consists in the ability to find the information needed to analyse, set up and solve the problem. It requires an a priori deep understanding of the situation and the ability of decoding the information addressed by the text (also the implied ones) expressed in various forms (linguistic, arithmetic, algebraic, graphic etc.). This entails the ability to know how to extract information from various representations expressed in several semiotic registers (Duval, 1993).

Once the "mathematical problem" is obtained, the solver proceeds with the *employment* of already known solution strategies or the elaboration of new ones, for example by applying facts, rules and algorithms; manipulating numbers, information, graphical data, expressions or equations, geometric constructions; using different representations and switching from one to another to get to the solution.[1] This second process takes place entirely in the world of mathematics and uses its language and methods. 1

The assessment of one (or more) solutions does not complete the cycle; in fact it requires to go through two further processes, although often in the didactic practice there is the tendency to underestimate this important phase of a posteriori analysis. The third process of the cycle *(interpret)* involves the students' ability to reflect on mathematical proceedings, solutions or conclusions and to interpret them in the context of the initial problem, thus requiring a deep understanding of the mathematical meaning of what has been obtained. In this process the students are particularly encouraged to formulate and communicate explanations and arguments related to the starting problem, belonging to a context of reality, reflecting on both the modelling process and the results obtained.[2] 2

In the fourth process *(evaluate)* is required the ability to evaluate the acceptability or not of the solving processes and of the solutions found on the basis of the real conditions posed by the problem. This involves a critical reflection on the possible limits or strengths of the mathematical model used, on why one or more solutions have been obtained, on their meaning in the specific context of the problem situation, as well as a wider consideration of how the real world affect the chosen mathematical model.

1. In the first two processes of the cycle we can also identify the *Exploring and trying* process contained in the *Piano di studio della scuola dell'obbligo ticinese* (DECS, 2015).
2. This cycle process is particularly linked to two of those provided for the *Piano di studio della scuola dell'obbligo ticinese* (DECS, 2015): *Interpret and reflect on results and Communicate and argue*.

In this cycle it is possible to highlight two kinds of mathematization defined by Treffers (1987) and then by Freudenthal (1991): one *horizontal* and one *vertical*. The following quote explains the distinction between the two:

> «Thus, through an empirical approach – observation, experimentation, inductive reasoning – the problem is transformed in such a way that it can be approached by strictly mathematical means. The attempt to schematize the problem mathematically is indicated by the term "horizontal" mathematization. (...) The activities that follow and that are related to the mathematical process, the solution of the problem, the generalization of the solution and the further formalization, can be described as "vertical" mathematization».
>
> (Treffers, 1987, p. 71)

In all phases of the mathematical activity both mathematizations complement each other (De Lange, 1987). In the initial definition of horizontal mathematization, the emphasis is on the transition from the real world to the mathematical world. Instead, the vertical mathematization refers to the process only within the mathematical world. However, in Jupri & Drijvers (2016) a wider interpretation is provided that can be applied to the cycle of mathematization proposed by PISA (OECD, 2013). The horizontal mathematization can be considered as the transition and communication between the two worlds (real and mathematical), while the vertical one as the elaboration of strategies and procedures within the same world (Figure 2).

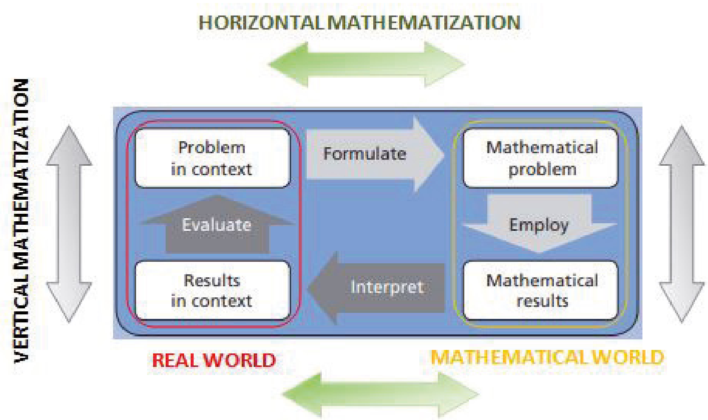


Figure 2
Horizontal and vertical mathematization in the mathematization cycle.

Horizontal mathematization requires both a mathematical translation of the real situation through semiotic representations (*formulate*, from the real world to the mathematical world), and an analysis and interpretation of the mathematical results obtained in the context of the real situation (*interpret*, from the mathematical world to the real world).
Vertical mathematization requires both a reorganization and reconstruction of the problem within mathematics – i.e. the manipulation of mathematical models, the use of procedures and concepts, recognizing recurring patterns and strategies to be used with known methods or to be explored (*employ*, within the mathematical world) – and the verification of the problem conditions, the generalization of the solving procedures and the recognition of a possible application of these procedures to similar problems (*evaluate*, within the real world) (Jupri & Drijvers, 2016).

Explaining these phases of mathematization allows us to analyse in depth the competences of the students and to foresee eventual actions of specific intervention in case of difficulty.

2.2 Some difficulties in the process of Mathematization and modelling

In literature there are several studies focused on the analysis of the students' difficulties in the process of *Mathematization and modelling*; in particular, they focus on one of the various phases described above and in different school levels (Jupri, Drijvers & Van den Heuvel-Panhuizen, 2014; Jupri & Drijvers, 2016; Wijaya, Van den Heuvel-Panhuizen, Doorman & Robitzsch, 2014; English & Watters, 2004b; Zan, 2007a; 2016; D'Amore, 2014).

In line with what was developed by Newman (1977) for the resolution of verbal problems (Newman Error Analysis), later taken up by Clements (1980), we can frame and categorize these difficulties in the following way:

– *Words' meaning.* In literature it is already known how many difficulties encountered in mathematics derive from linguistic deficiencies related in particular to the words' meaning, the so-called "dictionary" (Ferrari, 2003; Fornara & Sbaragli, 2013; Zan, 2016).

– *Understanding of the situation.* Difficulties in understanding the meaning of the problem and correctly representing the situation described in the text of the problem. According to some researches (D'Amore, 1996a; 2014; Zan, 2007a; 2016) students' difficulties often arise from issues related to the initial phase of understanding. The teachers themselves highlight the fact that often the child reads the text but does not understand it thoroughly, or does not capture it in a single whole. As D'Amore states:

> «a deficient or distorted mental representation of the problem is one of the most frequent causes of failure and it is therefore here that we need to intervene effectively and intelligently. (…) The child may experience difficulties in the first phase of the processes of symbolization: from the text to the mental image evoked by the text; [or] can imagine situations that cause conflicts between the image itself and the skills already possessed».
>
> (2014, p. 172, translation by the authors)

In fact, it seems often to be lacking an effective reconstruction of the problematic situation (Zan, 2011). According to the author, this lack generally comes from two phenomena: the difficulty in understanding and giving up understanding.

– *Transformation of text into a mathematical model.* Inability to translate the real situation into a mathematical problem, difficulties in establishing links between natural language and specific language of mathematics: verbal (where we use terms drawn from natural language, often with different disciplinary meanings), graphic, algebraic, symbolic, logical etc. Among the major difficulties that students face in solving problems is precisely the inability to manage different representations and to move from one to another in the modelling phase (Duval, 1993, D'Amore, 2006).

Moreover, even though research shows that elementary school children can engage in complex situations with adequate support and guidance from teachers, traditionally they are introduced to modelling only in middle school, preventing them from taking the first steps towards aspects involving this process (Diezmann, Watters & English, 2002; Doerr & English, 2003).

- *Mathematical resolution.* Difficulties related to algorithmic and conceptual learning (Fandiño Pinilla, 2008) employing applied mathematical procedures. This category includes, for example, calculation errors, algorithms and formulas.

- *Interpretation of results.* Difficulties in interpreting the mathematical solution in the real context and critically re-reading the results obtained. Students often misunderstand the meaning of the contextualized problem and provide mathematical solutions that are not consistent or relevant to the situation described in the task (Palm, 2008).

Clements' researches (1980) show how the failure of the students, who do not know how to solve problems, occurs in the first three points, which involve processes that are activated before the application of mathematical procedures. For this reason it is extremely important to focus on the teaching action in particular on the first steps of solving a problem.

2.3 Linguistic difficulties in understanding the text

In a standardized test, items related to the *Mathematizing and modelling* process can be considered *word problems.*

There are several definitions of the mathematical term *word problem*, in particular we choose to refer to that provided by Gerofsky (1996). According to the author, a mathematical word problem is a task presented through a text written in a verbal form, possibly integrated through mathematical symbolism. Frequently word problems also involve narrative aspects, as they describe plausible situations with characters who perform certain actions; in this case they are also called *story problems* (Verschaffel, Greer & De Corte, 2000).

The word problems proposed to the students are usually heteroposed; this means that the person who designs and administers them (usually the teacher or the experimenter) is not the same person who must solve them (usually the student) and often the aims of the people involved are not the same. This occurs in particular in standardized tests, where the intentions of the proposers and authors of the items may be very different from those of the students who have to solve them. In this case the items are presented through a common method of communication: a text. With this term we refer to any linguistic production of variable length (Ferrari, 2004). Studying the text of a task means considering different aspects; in particular, in this study we are interested in those related to the linguistic dimension.

It is not possible to think that the linguistic choices made during the formulation of the text of the problem do not have consequences on the interpretation of the task on the part of the solver. As Bara and Bucciarelli state (1992, p. 67, translation by the authors): «The manipulation of mental models is a cognitive process of particular importance in communication: the understanding of a linguistic statement requires the construction of a mental model starting from the wording itself». This process of text interpretation is therefore crucial when we analyse the behaviour of a solver dealing with a mathematical item, as this behaviour depends on the mental representation that the solver creates from the text. This mental representation has been described in terms of the *mental model* (Johnson-Laird, 1983) or *situation model* (Kintsch, 1988). Referring to the theory of mental models defined by Johnson-Laird (1983), Pancanti states:

> «the process of interpretation of a text occurs through three levels of representation: the first is the level of grapheme representation (the text itself); the second is the level of propositional representation; finally, the third level is precisely representation through a mental model. The propositional representation provides the meaning of the text obtained as a function of the meanings of the individual words and their syntactic relations. The mental model allows to identify the referents, their relationships and therefore a possible world with respect to what is described in the text itself».
>
> (2014, p. 109, translation by the authors)

Another perspective provided by Marini and Carlomagno (2004), based on a re-elaboration of the model described by Kintsch and Van Dijk (1978), affirms that the comprehension of a text is based on a preliminary abstraction of the meanings of the single words, afterwards of sentences (meanings organized in proportions) and their subsequent integration into conceptual networks that grow in complexity until they reach the model of the situation described by the text.

> «(...) Understanding a written text or a discourse or an oral conversation consists in establishing the connections between the basic linguistic structures that are gradually processed. These connections provide themselves consistency to the mental representations that the listeners/readers made from the text».
>
> (Marini & Carlomagno, 2004, p. 9, translation by the authors)

Therefore, it is clear that the first step for solving a task is strictly related to the interpretation of the text in order to make a model of the situation. As we have already anticipated, difficulties in this step are well known for several years; for example, Mayer (1982), De Corte and Verschaffel (1985), Laborde (1995), D'Amore (1996b; 1997a; 2014), Verschaffel et al. (2000), Ferrari (2004), Zan (2007b), Fornara and Sbaragli (2013) showed how difficulties observed in relation to the process of solving the word problems may be caused by an inadequate understanding and interpretation of the text with which the task is presented, in particular by the influence of the editorial variables of the text (lexical, syntactic, textual). As D'Amore states:

> «Often the text is not expressed in the language that the child expects or in his own language (...) and therefore the child must "translate" semantically from an adult language to his own language, understand the sense of the request, in order to create an image of what the problematic situation proposes. It is clear that students need a linguistic education at a non-trivial level (...)».
>
> (2014, p. 132, translation by the authors)

In particular, the student must know the words' meaning of the Italian language (specialized or common) presented in the text, the so-called *dictionary*; the student must then have an appropriate *encyclopaedia*, that is the knowledge of the world's things, which is necessary to master in order to grasp the many implicit terms present in the text (for further information refer to Zan, 2007b). This is an awkward question because, as Zan says:

> «(...) in front of a text written as a problem, the fact that children do not know the correct meaning of the words used does not necessarily imply that they are aware of it, and that they interrupt the interpretation process in the absence of such an information: when they encounter unknown words they sometimes readjust what they feel in a reasonable construction for them».
>
> (2016, p. 50, translation by the authors)

In a previous work, D'Amore (1997b) highlights the same aspect, through the request to solve a problem in which there was an invented word. At this request children tend to re-interpret the unknown word, giving it reliable semantic connotations with respect to the reality described by the text.

> «It is like triggering a didactic contract clause according to which it cannot happen that the teacher inserts a nonexistent word in the text. This is a clause belonging to the group that I like to call "trust in the teacher". It is rather plausible for the child that it is a word he does not know, but that certainly means something; which does not seem to prevent the resolution at all».
>
> (D'Amore, 1997b, p. 250, translation by the authors)

Also a research carried out within the *Italmatica* project by Fornara and Sbaragli (2013; 2016) highlighted the difficulties in understanding the text by primary school children, deriving from linguistic aspects, and the erroneous attitudes adopted by students in order to solve problems. The research aimed at investigating the strategies implemented by students engaged in solving two standard scholastic problems. Such problems were easy with regard to the *mathematical structure* (possible solving processes, type of numerical data etc.), but they were complex in terms of the *narrative structure*, on which the process of understanding – or representing – is based on. In particular, in these texts the solving process was bound to the correct interpretation of some words' meaning: that is, to the mastery of the dictionary, to which the encyclopedic knowledge is linked. The data collected revealed the erroneous attitude of the students when trying to find a solution even when the comprehension of the text was incomplete (verified through the request to write the meaning of some words), thus demonstrating that the need to provide the teacher a result is stronger than admitting that they do not possess all the linguistic-encyclopedic knowledge to satisfy the request of the problem.
In fact, several students were aware that they did not know the meaning of some words in the text, since they made it explicit (with assertions like "I do not know the meaning"), but this did not lead them to interrupt the resolution process. As Zan (2007b, p. 746, translation by the authors) states, «Of course, if the reader realizes that he does not know the meaning of a word, he can ask for it or look for it, or suspend the interpretation of the text. But this does not necessarily happen».

In a subsequent experimentation carried out with primary school students (Fornara & Sbaragli, in press), the researchers wanted to "break" this stereotypical habit linked to solving math problems, based on the belief that solving a problem should immediately follow its formulation, even in the absence of information useful to the goal. In particular, they wanted to raise awareness among the students about the importance of reflecting on the meaning of the words presented in the text and of the whole situation, showing students their relevance for solving a problem. This experimentation led to significant considerations by the students and an improvement in problem solving.

In addition to the linguistic aspects, among the several student's difficulties in understanding, there are others related to the meaning of the problem itself, i.e. concerning the type of situation in which the mathematical problem is contextualized and the link between the situation described and the question posed (Zan, 2016).

According to Zan (2012), the representation of the situation described in the problem's text is often circumvented by students in favour of "pathological behaviours", which have been long highlighted by the research in mathematics teaching, such as the *selective reading* of the text, i.e. the reading oriented to the search for numerical data and key words that suggest the way to combine them, the transcription of the result of an algorithm regardless of the starting context, which testifies to «an a priori surrender to understand, since the strategies used seem to disregard the comprehension of the text» (Zan, 2011, p. 18, translation by the authors). As Zan says:

> «The interpretation of this phenomenon is complex, and brings into play several interacting factors (for a synthesis see Verschaffel et al., 2000): stereotypes related to standard word problems, implicit and explicit norms that regulate mathematical activity in the classroom (the so-called didactic contract), the beliefs that children build by interpreting the activity with the problems».
>
> (2012, p. 437, translation by the authors)

As demonstrated by several researches in the field of mathematics education, natural language can therefore become an "extra" (and unavoidable) obstacle in the interpretation of a mathematical text and, if not adequately mastered, is likely to be one of the most pervasive obstacles to its resolution.

3 Methodology

First administration of the test. In May 2015, towards the end of the school year, a standardized mathematics test was given to all 3012 5^{th} grade students in Canton Ticino. The test consisted of two sets of 45 items each (below, each set is called a dossier). The students had one hour to solve the items in each dossier. The time interval in which the dossiers were administered was two weeks.

As already said, in the didactic evaluation of this test it was decided to analyse the process *Mathematizing and modelling*, consisting of 15 items related to the field *Numbers and Calculation* and 15 to the field *Quantities and Measures* (Sbaragli & Franchini, 2017). The items belonging to this category are of considerable interest, since they present a stimulus describing a realistic context on which the student must operate, in order to construct a model of the presented situation.

To perform a precise didactic evaluation of the students' answers, we analysed the protocols of a sample of 508 children randomly extracted from all the dossiers, in order to guarantee statistical validity. The sample was chosen in order to balance all the school districts of Canton Ticino and to be balanced by gender; in this way almost all schools were included.

This choice made it possible to identify not only the percentages of correct, incorrect and missing answers, but also the most frequent types of errors and to identify some interpretative hypotheses of the reasons that may have prompted the students to provide certain answers. To confirm these hypotheses, a second administration of some items was carried out, followed by interviews with some students.

Second administration of the selected items. To validate the hypotheses formulated by the answers to the standardized test, at the beginning of the 2016/2017 school year, 15 items were administered in the fields *Numbers and Calculation* and *Quantities and measurements*. We selected the items in which the answers provided did not explicitly presented the solving processes involved in determining the solution, and we proceeded to carry out individual interviews to investigate the process more in depth.[3] In order to collect data as similar as possible to those collected previously, it was necessary to identify a sample that was as similar as possible to that which had been given in the standardized test. Since this was the beginning of the school year, it was not possible to select a population of the same school level as the sample (5th grade students), because the topics dealt with in class and the school experience could not have reached the same standard. For this reason, a sample of 6th grade students was identified, for which it was possible to hypothesize a level similar to students at the end of 5th grade school. Eight classes of three secondary schools were involved: Locarno 1, Locarno 2 and Minusio for a total of 174 students.[4]
The dossier was composed of the 15 items selected; it was administered during the school hours and a time limit proportional to the time given to the whole standardized test (20 minutes) was provided to resolve them. Starting from the analysis of the protocols, some students were selected and interviewed. In Table 1 we present in detail the eight classes involved in the administration, specifying the number of students to whom the dossier was administered and the number of students interviewed.

3. We thank Romina Casamassa and Gemma Carotenuto for their help in carrying out the interviews.
4. We thank the secondary school directors of Locarno 1, Locarno 2 and Minusio for the availability provided: Daniele Bianchetti, Carla Stockar and Paolo Iaquinta and the mathematics teachers: Marco Banfi, Rocco Legato, Lara Caverzasio, Daniele Pezzi, Sara Cataldi and Daniele Zezza.

School	Class	N. of students to whom the dossier was administered	N. of students interviewed
Locarno 1	IA	20	9
	IC	19	10
Locarno 2	IA	24	12
	IB	20	8
	IC	21	14
Minusio	IA	22	7
	IB	23	12
	IC	23	12

Table 1
Number of students involved in the experimentation.

In this paper we choose to report the analysis carried out on a single item of the *Numbers and calculation* area:

Figure 3	Figure 4
Un appartamento aveva 7 locali.	An apartment had 7 rooms.
Dal locale più grande sono state ricavate 2 camere.	Later, 2 bedrooms were obtained from the largest room.
Quanti locali ha ora l'appartamento?	Now how many rooms does the apartment have?
Risposta:	Answer: ....

Figure 3
Original item administered in the Italian language.

Figure 4
English traduction of the item.

This is an item that deals with a contextualized problem; the text describes an apartment initially composed of 7 rooms to which a change was made: the largest room was divided into two further bedrooms in order to add a new room to the apartment, so the number of rooms in the apartment has passed from 7 to 8.

The item is presented through a text written in a verbal form, it is necessary to decode the text information to understand the situation, also from a linguistic point of view. Analysing the item, the process of *horizontal mathematization*, that is the translation of the real problem in the mathematical problem, can generate difficulties for students because it involves the passage and the communication between the two worlds expressed through different registers.

Moreover, in the Italian version of the text, two different words were used to write the term "room": *locali (rooms)* and *camere (bedrooms)*. As we will see below, this lexical choice has caused difficulties in understanding the text. In the following, we will keep the distinction using these two terms and their Italian traduction.

4 Analysis of the results

4.1 Analysis of the results of the first administration

The item we study is particularly interesting to be analysed. We report below the results obtained from the analysis of the 508 protocols related to the standardized test of the selected sample:

N. of students and percentage of sample answers (May 2015)		
	N. students	**Percentage**
Correct answers	180	35,4%
Wrong answers	268	52,8%
Missing answers	50	9,8%
Absent students	10	2%

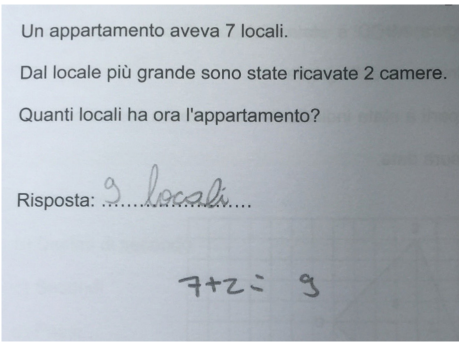
Table 2
Percentage of sample answers.

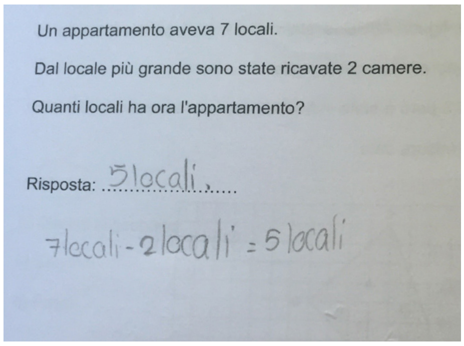
The percentage of wrong answers of this item is more than 50% of the answers provided. This is the only item of the test among those open-ended that has collected such a large number of failures. Moreover, the percentage of missing answers is low with respect to the number of incorrect answers and compared to those of other items in the dossiers (Sbaragli & Franchini, 2017). This could point out that the item was not perceived by students as particularly difficult. The table, shows the most frequent incorrect answers provided by the students:

Categories of wrong answers	**N. students of the sample (May 2015)**	**Percentage of the sample (May 2015)**
9; 9 locali o 9 camere *(9; 9 rooms or 9 bedrooms)*	124	24,4%
5; 5 locali o 5 camere *(5; 5 rooms or 5 bedrooms)*	87	17,1%
6; 6 locali o 6 camere *(6; 6 rooms or 6 bedrooms)*	28	5,5%
7; 7 locali o 7 camere *(7; 7 rooms or 7 bedrooms)*	12	2,4%
Others	17	3,4%
Total	268	52,8%

Table 3
Categories of wrong answers provided by students of the sample (May 2015).

Only in a few cases it was possible to detect, besides the result, also the solving process adopted by the student to provide the answer, i.e. the procedure that allowed to identify the numerical value presented in the response.





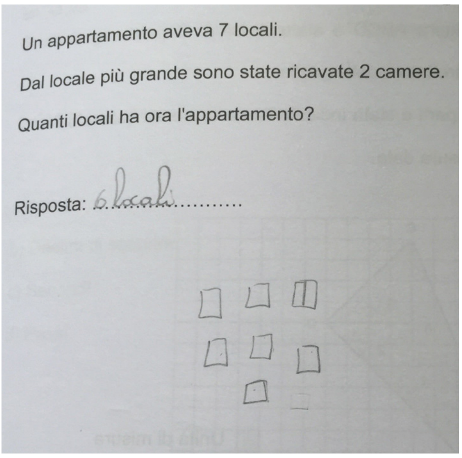


Figure 3, Figure 4, Figure 5
Examples of wrong answers in which the solving process is explained.

In **Figure 5**, the student answers *9 locali (rooms)*. This value is calculated through the sum: 7 + 2. This choice could suggest that the student interprets the represented situation in the text as adding 2 further *locali (rooms)* to the already existing 7. In a different way, the student who provided the answer presented in **Figure 6** could have interpreted the situation as if two *locali (rooms)* had been eliminated from the apartment, calculating that the number of *locali (rooms)* actually left was 7 - 2 = 5. The answer shown in **Figure 7** is again different, the student uses an iconographic representation to model the situation; this model is consistent with the situation presented but the answer provided is not.

Based on the answers given to this item, some interpretative hypotheses were made: students' wrong answers could mainly be linked to linguistic aspects of the text that condition the horizontal mathematization. In particular, they could be linked to the *formulation* process, which requires students to extract from the written text the necessary information to analyse, set up and solve the problem.

In the item there are in fact some linguistic aspects that could have had an effect on the decision-making of the students involved. In particular, in the Italian version of the text (**Figure 3**), the authors of the item chose to use two terms, *locale (room)* and *camera (bedroom)*, to avoid repetition within the text. This means that the two distinct terms must be interpreted by the solver as synonyms. In fact, in the Italian dictionary of synonyms and antonyms (Treccani, 2003), the term *locale (room)* is defined as follows:

> ***Locale (room):*** *[…] space or complex of interior spaces in building constructions: the apartment has four rooms ~ bedroom (Treccani, 2003, term locale, translation by the authors).*[5] 5

Similarly, the term *camera (bedroom)* is defined as follows in the dictionary of the Italian language:

> ***Càmera (bedroom):*** *[…] generally speaking, any interior space of a residential building that does not have a specific purpose due to its particular shape and size. More concretely, each of the rooms (locali) that comprise an apartment: bedroom, dining room, living room […] (Treccani, 2003, term camera, translation by the authors).*[6] 6

In the dictionary's definition of the term *camera (bedroom)* the term *locale (room)* is used and specified as a synonym. However, if we look for the definition of the word *locale (room)* in a dictionary of the Italian language, we can see that this term can refer to a more generic space or a complex of interior spaces with respect to the word *camera (bedroom)*, in fact:

> ***Locale (room):*** *[…] space or complex of interior spaces, even in non-building constructions (such as, for example, in ships), which by shape, structure, equipment is destined for certain uses: a country in which school rooms (locali) are scarce; look for a suitable venue (locali) for a conference, for a meeting; the place (locale) is too small to be used as a cinema; in ships, machine rooms, boiler rooms, etc. More generally, space, especially of public buildings: barracks, colleges with large and airy rooms, etc. (Treccani, 2003, term locale, translation by authors).*[7] 7

This is a very subtle distinction that could however lead to different interpretations of the text compared to the intentions of the author of the item: the term *locale (room)* indicates a space in which several *camere (bedrooms)* can appear. With reference to this interpretation, the situation presented in the item could be understood by some students as a circumstance in which the number of *camere (bedrooms)* in the apartment varies, especially increases, but the number of *locali (rooms)* remains unchanged, equal to 7.
Instead, other students may have interpreted the situation as the transformation of

5. Locale s. m. [dal fr. local, uso sost. Dell'agg. local "locale"]. - 1. a. [ambiente o complesso di ambienti di costruzioni edilizie: la casa ha quattro l.] ≈ camera, stanza, vano. (Treccani, 2003, voce locale).
6. Càmera s. f. [lat. camĕra, camăra «volta, soffitto a volta di una stanza», dal gr. μάρα]. – 1. a. In senso generico, qualunque ambiente interno di un edificio per abitazione, che non abbia, per particolarità di forma, dimensioni e impianti, una destinazione speciale. Più concretam., ciascuno dei locali che compongono un appartamento: c. da letto, c. da pranzo, c. da soggiorno; un appartamento di tre c. e cucina. (Treccani, 2003, voce camera).
7. Locale s. m. [dal fr. local, uso sostantivato dell'agg. local «locale»]. – Ambiente o complesso di ambienti, anche in costruzioni non edilizie (come, per es., nelle navi), che per forma, disposizione, attrezzatura, e sim., è destinato a determinati usi: paese in cui scarseggiano l. scolastici; cercare un l. adatto per una conferenza, per una riunione; il l. è troppo ristretto per essere adibito a sala cinematografica; nelle navi, l. macchine, l. caldaie, ecc. Più genericam., stanza, ambiente, soprattutto di edifici pubblici o per collettività: caserma, collegio con l. ampî, ariosi, ecc. (Treccani, 2003, voce locale).

a *locale (room)* into two *camere (bedrooms)*. In this way, the largest *locale (room)* is no longer included in the *locali (rooms)* count and for this reason *locali (rooms)* are reduced to 6 (in line with the response presented in Figure 7).

A further term that may not belong to the student's dictionary is the verb *ricavare (obtain)*. In the intent of the author, this term is presented with the meaning of obtaining. In fact, always in a vocabulary of the Italian language, the term is defined as:

> ***Ricavare (obtain):*** *[...] obtain, deduce or extract, usually through a more or less profound elaboration or transformation: to obtain a ladder in the rock; the statuary group is obtained from a single block of marble; brandy is obtained from the distillation of the marc; a very rich subsoil from which iron and coal are obtained. (Treccani, 2003, term ricavare, translation by the authors).*[8]

If this term does not belong to the dictionary of the students involved in resolving the item, it is possible that they react by not answering the question of the item or interpreting the meaning of the verb in a personal way.

Analysing students' solution processes, it is possible that the term *ricavare (obtain)* could be interpreted as: add, remove, unite. Consistently with each of these interpretations, students may provided an answer to the item different by the correct one. For example, a student could interpret the verb as a synonymous with *to add* and provide the answer 9, calculating, 7 + 2 = 9 (Figure 5); or, the one who to the word *ricavare (obtain)* associates the meaning of *removing*, could indicate as a response 5 rooms and therefore perform the calculation: 7 - 2 = 5 (Figure 6). Finally, the student who interprets the verb as a synonymous with *to put together* could provide the answer 6 because two rooms have become one and therefore 7 - 1 = 6.

Moreover, the context described by the text of the item implicitly refers to construction works carried out in an apartment, a context that may not be included in the facts related to the experience of the students.
In order to demonstrate the truthfulness of these hypotheses and to establish if these answers are actually connected to the incorrect interpretation of the text or to other phases of the *Mathematization and modelling process*, it is necessary to deepen our analyses through the second administration.

4.2 Analysis of the results of the second administration

The analysis of the responses obtained from the second administration of this item, carried out with 174 6th grade students in September 2016, highlights a uniformity with the responses compared to those collected in the cantonal survey with 5th grade students in May 2015 (Table 4). In fact, even in the second case we can see that the percentage of students who answered correctly remains around 30%, even if lower

8. Ricavare v. tr. [comp. di ri- e cavare]. 2. Cavare fuori, ottenere, trarre o estrarre, di solito attraverso una elaborazione o trasformazione più o meno profonda: r. una scala nella roccia; il gruppo statuario è ricavato da un unico blocco di marmo; acquavite ricavata dalla distillazione delle vinacce; un sottosuolo ricchissimo da cui si ricavano ferro e carbone. (Treccani, 2003, voce ricavare).

than the previous one, and the percentage of incorrect answers is around 55%. In this second administration, however, unlike the previous case, we asked students to indicate the reason for a possible lack of response. As we can see from the data collected, about 5% of students explicitly did not respond due to a lack of understanding of the text of the item.

	Percentage first administration (May 2015)	Percentage second administration (September 2016)
Correct answers	35,4%	29,9%
Wrong answers	52,8%	55,2%
Missing answers without justification	9,8%	9,8%
Missing answers with justification linked with a lack of text comprehension	-	5,1%
Absent students	2%	-

Table 4
Percentage of answers of second administration compared with the first.

Also the wrong answers provided by the students in this administration can be identified through the same categories observed in the previous survey (Table 5).

Categories of wrong answers	Percentage first administration (May 2015)	Percentage second administration (September 2016)	N. of student of second administration	N. of students interviewed
5; 5 locali o 5 camere *(5; 5 rooms or 5 bedrooms)*	17,1%	20,7%	36	9
9; 9 locali o 9 camere *(9; 9 rooms or 9 bedrooms)*	24,4%	16,1%	28	8
6; 6 locali o 6 camere *(6; 6 rooms or 6 bedrooms)*	5,5%	9,2%	16	7
7; 7 locali o 7 camere *(7; 7 rooms or 7 bedrooms)*	2,4%	4%	7	3
Others	3,4%	5,2%	9	5
Total	52,8%	55,2%	96	32

Table 5
Percentages of answers sorted by categories of incorrect answers provided by the students in the second administration compared to the first.

Comparing the percentages related to the different types of responses, we can observe that in general the results collected from the second administration are not very different from those collected in the first. Furthermore, thanks to the interviews we are able to confirm or not the interpretative hypotheses presented in paragraph 4.1. In particular, in the following, we will discuss each individual category of response by providing a posteriori interpretation based on the interviews carried out.

Students who answer 5

The interviewed students who answered 5 motivate their choices in various ways that can be distinguished in the following two categories:

– some students explicitly declare that they consider the verb ricavare *(obtain)* with the meaning of *demolish/remove*;
– others show various difficulties in understanding the text of the item, declaring that they cannot interpret the situation, without making explicit an alternative meaning for the verb.

Hence we found difficulties in horizontal mathematization, that is in the transition from the real situation to the mathematical model that interprets it. In some cases, the choice of the applied mathematical operation seems totally disconnected from the situation described by students, that is the interpretation of the building action carried out on the rooms of the apartment. Some students make a *selective reading* of the text identifying the data in the text and choosing the operation to be performed, without seeking a link between the two worlds (real and mathematical) (Zan, 2011;D'Amore, 2014).
Below we present the transcripts of some parts of the interviews carried out in order to clarify the types of motivations described above (Table 6), with the initial I. we indicate the interviewer, the other letters refer to the students' initials.

Interview transcript	Explanation
I.: "What do you do in this apartment?" L.: "You get two more rooms." I.: "Practically what do you do?" L.: (after a few seconds of silence) "I do not know." I.: "Is the text unclear?" L.: "Yes." I.: "Can you imagine what the workers who come into this apartment have to do?" L.: "They have to demolish."	The student L. is asked to describe the situation; initially he repeats the sentence presented by the text. After a more explicit request by the interviewer, he declares that he has not understood. Following a second request for clarification, the student reveals his interpretation of the term *ricavare (obtain)* as to *demolish*. In this case the student belongs to the first type of motivation provided.
I.: "Do you remember the *locali (rooms)* item? Can you show me what you did to answer?" M.: "I did 7-2." I.: "You did 7-2, but what does this question ask you? Can you tell me about it?" M.: "Mmm, from the largest room have been *ricavati (obtained)*; two rooms have been removed, I think." I.: "So the term *ricavare (obtain)* for you means removed?" M.: "Yes."	Following the interviewer's question, the student M. explained the operation performed to determine the answer provided. Subsequently, the interviewer asks the student to explain his interpretation of the situation, and he declares to interpret the meaning of the verb *ricavare (obtain)* as a synonym to *remove*. Also in this case the student falls into the first category of motivations.

I.: "What do you think about this here?" (I. indicates the item). R.: "I did not understand it well. There are 7 *locali (rooms)*, two *camere (bedrooms)* have been obtained from the largest *locale (room)*." I.: "What does this mean?" R.: "It means that a *locale (room)* has two *camere (bedrooms)*, the others have 1." I.: "So from a *locale (room)* you get two *camere (bedrooms)*, what does that mean?" R.: "Eh, I do not know, from the largest *locale (room)* have been obtained two *camere (bedrooms)* so you can go to sleep in 2 and in the other in 1." I.: "But beds or rooms?" R.: "Ah, but *camere (bedrooms)* meaning *stanze (rooms)* (other Italian synonym for rooms)?" I.: "Yes." R.: "Ah, now I understand! The *locale (room)* is divided into *stanze (rooms)*." I.: "They divide it in how many *stanze (rooms)*?". R.: "Two, that is like making a wall in the middle."	The student R. explains the lack of understanding of the text, mainly linked to the interpretation of the real situation. In this case the student does not attribute to the term *ricavare (obtain)* an alternative meaning, falling into the second category. The difficulty of the student is revealed in the understanding of the terms *camere (bedrooms)* and *locali (rooms)*; he tries to overcome such difficulties introducing another Italian synonym for the term *stanza (room)*.

Table 6
Transcript of part of the interview to students who answered 5.

Students who answer 9

Many students who answered 9 *locali (rooms)*, explain the operation carried out: 7+2=9. Also in this case, two different reasons emerge from the interviews:

– those who attribute to the term *ricavare (obtain)* the meaning of adding;
– those who show a difficulty in creating a mathematical model that allows to represent reality.

As in the previous case, it emerges a linguistic difficulty linked to the meaning of the term *ricavare (obtain)*, interpreted differently from the previous case, and one linked to the modelling of the situation. In the latter case, the students show that they understand the type of building intervention done on the apartment, but they respond quickly looking for an operation suitable to model this intervention in reference to the numerical data presented, without interpreting in reality if a room was already counted. Table 7 shows two examples of the two reasons found among the interviewed students.

Interview transcript	Explanation
I.: "Can you show me how did you answer?" L.:"*Ricavate (Obtained)*? Eh, I did not remember well what *ricavate (obtained)* means" I.: "In your opinion what does this mean? Let's try to read again." L. reads aloud. I.: "So, you answered 9, can you explain to me how did you get 9?" L.:"Maybe I did a stupid thing but I thought, adding two *camere (bedrooms)*. Then it was added to the apartment." I.: "So you did?" L.:"Plus 2!"	In this case the student L. identifies with the first category, in fact, he explicitly does not remember the meaning of the term *ricavare (obtain)*. After a second reading, suggested by the interviewer, the student explains that he chose to perform the operation 7 + 2 because he interpreted the meaning of the verb *ricavare (obtain)* as adding.

R.: "The *locale (room)* is divided into *stanze (rooms)*." I.: "They divide it in how many *stanze (rooms)*?" R.: "Two, for example, they make a wall in the middle." R.: "He had 7 before and now he has 9!" I.: "9?" R.: "Because you have 7 and add 2." I.: "So you create two *stanze (rooms)*." R.: "No, you start with something you already had." I.: "Before what did you have?" R.: "A *stanza (room)*." I.: "And now?" R.: "2." I.: "So how many *stanze (rooms)* did you have?" R.: "Ah, 8!"	In this case the student R. makes a correct interpretation of the text; nevertheless he shapes the situation as if two rooms had been added and not one. In this case the student can be described with the second category. Only after the interviewer directs the reasoning, the student answers the question correctly by adding a single room to the 7 already existing.

Table 7
Transcript of part of the interview to students who answered 9.

Students who answer 6

Students who answered 6 highlight two different difficulties related both to dictionary knowledge:

– students who interpret the term *ricavare (obtain)* as to unite, and therefore two of the 7 rooms become one, thus the total number of rooms drops to 6.
– students who do not recognize the same meaning for the terms *locali (rooms)* and *camere (bedrooms)*, interpreting the stimulus as if one of the 7 *locali (rooms)* were changed into two *camere (bedrooms)*. In this way, 6 *locali (rooms)* and 2 *camere (bedrooms)* remain in the apartment after the building modification.

Table 8 shows two example interviews of these categories.

Interview transcript	Explanation
I.: "Let's look at this one here; you say that the apartment has 6 rooms, how did you find this answer?" C.: "So, an apartment has 7 *locali (rooms)* in the sense …" I.: "What does it mean *locali (rooms)* in your opinion?" C.: "It is like saying *camere (bedrooms)*." I.: "Ah, ok, so *camere (bedrooms)* and *locali (rooms)* have the same meaning?" C.: "Yes, and then, uh, two *camere (bedrooms)* have been *ricavate (obtained)* from the largest *locale (room)*, so I have put two *camere (bedrooms)* together." I.: "So *ricavare (obtain)* what it means?" C.: "Unite, two become one!"	In this case the student C. correctly interpreted the meaning of the words *locali (rooms)* and *camere (bedrooms)* but did not understand the meaning of the verb *ricavare (obtain)*. As it can be guessed from the transcription, in fact, for the student two rooms have been united and therefore two of the 7 have become one. In this case the student belongs to the first category described above.
I.: "How did you answer this question?". A.: "Here the text said 7 *locali (rooms)* and I thought that they eliminated one even if it was big they took it away." I.: "Ah, they took it away, where is it said?" A.: "It is said in the second, starting from a *locale (room)*, two rooms have been *ricavate (obtained)*."	In the first part of the interview, it may seem that the student had difficulty interpreting the term *ricavare (obtain)*. Actually, the questions of the interviewer allowed to show that the student understood the meaning of the term *ricavare (obtain)*, but did not grasp that the choice of using the word *camera (bedroom)* was made only to

I.: "What do you mean two rooms have been *ricavate (obtained)*?" A.: "That a *locale (room)* has been divided to make two *camere (bedrooms)*; so they have eliminated one that has become two *camere (bedrooms)* and therefore there are 6 *locali (rooms)* left."	avoid repeating the word *locale (room)*. In this case, in fact, the student explicitly understands that a *locale (room)* has been converted into two *camere (bedrooms)* but when counting the number of *locali (rooms)* he did not count the two new *camere (bedrooms)*, but only the remaining *locali (rooms)* that have not undergone any changes.

Table 8
Transcript of part of the interview to students who answered 6.

Students who answer 7

From the interviews it emerges that all the students that answer 7 do not consider the terms *camere (bedrooms)* and *locali (rooms)* as synonyms. In this case, therefore, there is a difficulty in the linguistic interpretation of the represented situation; students consider that the number of *locali (rooms)* remains the same and the number of *camere (bedrooms)* changes. Some examples are given in Table 9.

Interview transcript	Explanation
I.: "Do you remember this item?" F.: "Yes, I did it in my head.." I.: "Ok, now let me see how you did find the answer." F.: (F. reads the text aloud) "Eh, they are seven in any case." I.: "Show me how you did it, you can write it or say it in words." F.: "From the largest *locale (room)*, two *camere (bedrooms)* have been obtained, but are they *camere (bedrooms)* for the apartment?" I.: "So? There is a situation that speaks of *locali (rooms)*, *camere (bedrooms)* and apartments." F.: "I wrote 7 because I did not understand if the terms *locali (rooms)* and *camere (bedrooms)* were the same. That is as if the large *locale (room)* was divided into two?" I.: "Yes, right!" F.: "Ah! I only thought that two *camere (bedrooms)* had been created." I.: "Do you think *camere (bedrooms)* and *locali (rooms)* mean the same thing or not?" F.: "It depends on whether you mean a bedroom or space or a room in the sense of a big space from which you get a room that means you put the bedroom, the kitchen and the bathroom." I.: "So *camera (bedroom)* and *locale (room)* is the same thing or not in your opinion?" F.: "Maybe here is the same thing but I did not thought so."	In this discussion, the student F. clearly explains his difficulty in interpreting the terms *locali (rooms)* and *camere (bedrooms)*, to be considered as synonyms. His interpretation of the term *locali (rooms)* is broader, as indeed the term is also described by the dictionary of the Italian language. Only at the conclusion of the discussion with the interviewer he states that in the item perhaps the two terms have been used as synonyms.
I.: "Why did you answer 7 *locali (rooms)*?" A.: "Because if there are 7 *locali (rooms)* and the biggest one gets 2, the number remains the same." I.: "But what is a *locale (room)*?" A.:"A *stanza (room)*." I.: "And the *camera (bedroom)*?" A.:"It's also a *locale (room)*." I.: "So *camere (bedrooms)* and *locali (rooms)* are the same thing?" A.:"No, the *locale (room)* is an apartment."	Student A. shows that, regardless of the operation done on the *camere (bedrooms)*, the number of *locali (rooms)* remains unchanged. The student explicits confusion in the interpretation of the term *locali (rooms)*, consequently a failure to understand the equivalence of the meaning of *camere (bedrooms)* and *locali (rooms)* in the text of the problem.

Table 9
Transcript of part of the interview to students who answered 7.

Students who answer with other values

Among the different responses from those hypothesized before the administration, it should be noted: 14 *locali (rooms)*; 5 of 9 students chose this answer.

From the transcripts presented below (Table 10), the students interpreted the situation as if two *camere (bedrooms)* had been obtained from each *locale (room)*. This aspect shows that the students did not pay attention to the fact that the text only talks about the largest *locale (room)* and therefore they interpreted the text as if it were: "an apartment has 7 *locali (rooms)*; from each *locale (room)* two *camere (bedrooms)* have been created."

This highlights how often the students read the text of a problem superficially, without giving importance to the different words, but hastily deducing what they think the text wants to communicate.

Interview transcript	Explanation
I.: "Explain this item of the apartment" R.: "I calculated that there are two *camere (bedrooms)* and each *camera (bedroom)* has 7 *locali (rooms)* then I did 7 + 7 which is 14."	Asked to explain the resolution, student R. explicits without hesitation her interpretation: from every *locale (room)*, two *camere (bedrooms)* have been created.
I.: "Tell me about the situation described in the item, what it was asking for." S.: "An apartment has 7 *locali (rooms)*." I.: "What does this mean?" S.: "That the apartment has 7 *cam…ere (bed… rooms)*." I.: "It has 7 *camere (bedrooms)* and then?" S.: "Two *camere (bedrooms)* have been made from the largest *locale (room)*." I.: "And what does this mean?" S.: (silence) "That in the bigger *locale (room)* they added two *camere (bedrooms)*." I.: "How did you answer 14?" S.: "7 × 2." I.: "Why did you do 7 × 2?" S.: "Because before he had 7 *locali (rooms)* and then he said he had two more." I.: "From all *locali (rooms)*?" S.: "Yes."	In this second example, instead, the student S. shows a certain hesitation. In this case, he has clearly understood that *camere (bedrooms)* and *locali (rooms)* are terms that must be understood as synonyms, but he has not paid attention to the fact that two *camere (bedrooms)* have been created only from one of the 7 *locali (rooms)* and not by all of them.

Table 10
Transcript of part of the interview to students who answered 14.

This attitude is also linked to the frequent phenomenon of *selective reading* of the text; in fact, the students hastily read the text, focusing only on the information presented through the numerical data:

– the apartment has 7 *locali (rooms)*;
– a *locale (room)* has been converted into 2 *camere (bedrooms)*.

Students may have superficially interpreted the other information that has been proposed through verbal and non-numerical language: "from the largest space".

The answers of the other 4 students were: 8, 5, 12 and in two cases 3,5. The reasoning behind this last answer is that a *locale (room)* has been divided into two, so the number 7 has been divided by two, obtaining 3,5.

5 Conclusions

Starting with the results obtained from the administration of this item, it is noted that often the difficulties in solving a problem are related to the first process of the cycle of mathematization, that is the *formulation*, which requires an interpretation and a deep understanding of the situation and the information needed to solve the problem, identifying the relevant mathematical aspects in order to find a model that interprets the situation. The evidence is therefore in line with Clements' considerations (1980). In fact, the failure in the solution of the item proposed in this paper occurs in the first three phases of the resolution process, and originates from difficulties related to the words' meaning, to the understanding of the situation and to the transformation of the text into a mathematical model. In this work it is interesting to observe how an aspect already highlighted in literature since the '80 can be amplified thanks to the standardized tests, which allow us to understand how many behaviours of the students are not random or related to the answers of the individual, but hide instead obstacles of different nature, profound and widespread. These evidences allow the teacher to focus on aspects of teaching/learning that can be improved or that may suggest targeted measures on students' learnings.

In this perspective, the items presented in the standardized tests relating to the aspect of competence *Mathematizing and modelling* (Sbaragli & Franchini, 2017) can be a reference point for teachers to identify possible difficulties of students in the different phases of the resolution process and for the development of significant educational paths.

In particular, from the answers given to this item it emerged that most of the students' difficulties were related to linguistic aspects of dictionary knowledge, or more generally of the encyclopedia, i.e. the facts of the world regarding the situation. The text, which could appear a priori understandable by students of this age, was actually very complex, even for the underlying linguistic implications. As D'Amore states:

> «To read the text of the problem, before solving it, seems to be an obvious preliminary condition. But it's not that easy. We have repeatedly seen (...) that there are obstacles to reading and understanding, there are mental images that are formed immediately and can be distracting, there are embarrassing terms, operations induced semantically, etc.».
>
> (2014, p.186, translation by the authors)

Moreover, on several occasions a selective reading of the text by the students emerged, that is, a reading oriented to the search for "alternative" solutions for understanding. With regard to this phenomenon, Sowder (1989) lists a variety of approaches, already mentioned in this text, which are often practiced by students in solving problems: trying to guess the operation; look at the numbers, and from those go back to the "right" operation; try all the operations and choose the one that gives the most "reasonable" answer; search for "keywords" and others.

It is therefore very important from an educational point of view to combine two disciplines traditionally felt very distant from each other, Italian and mathematics, with the aim of simultaneously developing mathematical and linguistic skills, and fostering in the students a productive attitude in facing problem solving, which deals with the need to understand and interpret the given situation (Demartini, Fornara & Sbaragli, 2017; Demartini & Sbaragli, 2015a; 2015b; Fornara & Sbaragli, 2013; 2016). Mathematical word problems text analysis carried out in class with the students, based on the deepening of the lexical aspects and on the interpretation of the situations described in relation to the experience of the students with particular attention to the modelling of the situation, are fundamental elements. These activities have the aim of developing and strengthening cognitive and metacognitive, meaningful and effective strategies for solving word problems.

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Authors / Elena Franchini*, Alice Lemmo° and Silvia Sbaragli*
*Dipartimento Formazione e Apprendimento – SUPSI, Locarno
°Università di Palermo
elena.franchini@supsi.ch, alice.lemmo@gmail.com and silvia.sbaragli@supsi.ch