The crucial role of narrative thought in understanding story problems[1]

Il ruolo cruciale del pensiero narrativo nella comprensione dei problemi

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Abstract / Within the solution of a mathematical word problem, the preliminary understanding process is viewed as crucial. This process shows critical points both in the case of a rather poor text of the problem and in the case of a very rich text, as shown by many answers given by students and reported in the literature, characterized by an apparent "suspension of sense-making". In this communication we propose an interpretation of this phenomenon, based upon the interplay between what Bruner calls the "narrative" and "logical" modes of thought.

Keywords: word problem; story problem; representation; solution; narrative thought; logical thought; sense-making.

Sunto / Nell'attività di soluzione di un problema matematico espresso attraverso un testo, è cruciale il processo preliminare di comprensione. Questo processo mostra criticità sia nel caso di un testo del problema molto sintetico, sia nel caso di un testo molto ricco, come dimostrano molte risposte date dagli studenti e riportate in letteratura, caratterizzate da un'apparente "sospensione di senso". In questo articolo proponiamo un'interpretazione di questo fenomeno, basata sull'interazione tra quelli che Bruner definisce modi di pensiero "narrativo" e "logico".

Parole chiave: word problem; story problem; rappresentazione; risoluzione; pensiero narrativo; pensiero logico; sense-making.

1 Introduction

Mathematical word problems deserve an important role in mathematics teaching, mainly at primary school.

Even though "word problem" literarily means only "problem expressed through a text", in mathematics education the term usually stands for:

> «a text (typically containing quantitative information) that describes a situation assumed familiar to the reader and poses a quantitative question, an answer to which can be derived by mathematical operations performed on the data provided in the text, or otherwise inferred.»
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> (Greer, Verschaffel & De Corte, 2002, p. 271)

The "situation assumed familiar to the reader" is often called "context" or "story", and in fact the expressions *word problem and story problem* are by most researchers

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explicitly used as synonyms.[2]

The international literature on word problems offers several examples of students' behaviours which suggest an apparent "suspension of sense-making" (Schoenfeld, 1991). Trying to interpret these behaviours research moved on along different lines. The studies carried out so far (for a survey see Verschaffel et al., 2000) on the one hand led to identify in the suspension of sense-making the responsibility of the scarce realism of word problems, of the stereotypical nature of the text and of the implicit and explicit norms which govern the problem solving activity (the "didactical contract"). On the other hand, they led to highlight the fact that many of the difficulties met by students lie in the preliminary phase of the construction of an adequate representation of the problem situation, and this makes it difficult to detect possible difficulties in the solution phase.

2 Understanding (story) problems

In the literature, studies on the nature of problems highlight in particular that:

«Although the numerical tasks are embedded in a context, the steryotyped nature of these contexts, **the lack of lively and interesting information about the contexts, and the nature of the questions asked at the end of the word problems** jointly contribute to children not being motivated and stimulated to pay attention to, and reflect upon, (the specific aspects of) that context.»

(Verschaffel et al., 2000, pp. 68-69, emphasis added)

Nesher (1980) underlines:

«The student who receives the well-defined SCH-PROB [school problem] does not have to be engaged in qualitative and quantitative decisions (...). In most cases, however, he is not able, because of the condensed style, to reconstruct the context from which the data was taken. In short, he **is not able to imagine the domain of objects and transformations that the author had in mind**. Instead he develops another strategy. He tries from the verbal formulation of the SCH-PROB text to infer directly the needed mathematical operation.»

(Nesher, 1980, p. 46, emphasis added)

The inference of the mathematical operation(s) from the verbal formulation of the problem may occur in a variety of ways, among which Sowder (1989) identifies the following: look at the numbers (they will "tell" you which operation to use); try all the operations and choose the most reasonable answer; look for isolated "key" words to tell which operation to use; decide whether the answer should be larger or smaller than the given numbers (if larger, try both addition and multiplication and

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2. For example Verschaffel et al. write (2000, p. ix): «word problems (or "verbal problems" or "story problems")», and Gerofsky (1996, p. 36): «Mathematical word problems, or "story problems", (...)».

choose the more reasonable answer; if smaller, try both subtraction and division and choose the more reasonable). Few students, even capable ones, give evidence of using the "mature" strategy "Choose the operation whose meaning fits the story". In other words, few students base their problem solving processes upon a representation of the problem.

Sowder notes that students who use the strategies listed above will be successful on many, if not most, one-step story problems in the whole-number curriculum. Also Gerofsky (1996) claims that often the process of representation of the problem is not essential to get the correct solution of a word problem. Looking at word problems as a linguistic and literary genre, she identifyes a three-component structure:

1) a "set up" component, establishing the characters and location of the putative story;
2) an "information" component, which gives the information needed to solve the problem;
3) a question.

Gerofsky writes:

> «(...) component 1 of a typical word problem is simply an alibi, the only nod toward "story" in a story problem. It sets up a situation for a group of characters, places and objects that is generally irrelevant to the writing and solving of the arithmetic or algebraic problem embedded in the later component. In fact, **too much attention to the story will distract students from the translation task at hand, leading them to consider "extraneous" factors from the story** rather than concentrating on extracting variables and operations from the more mathematically-salient components 2 and 3.»
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> (Gerofsky, 1996, p. 37, emphasis added)

Verschaffel et al. (2000) observe that the description made by Gerofsky of word problem solving is

a description of bad word problem solving – that «(...) bypasses the situation model and goes directly through some superficial cues from the text to the mathematical model.» (Verschaffel et al., 2000, p. 147).

De Corte and Verschaffel (1985), referring to the huge amount of data collected in their work with beginning first graders, claim that the lacking construction of an appropriate mental representation of the word problem is actually an obstacle to correct solution processes, and enables the researcher to understand some apparently absurd answers given by students. According to the American psychologist Richard Mayer (1982) one of the major contributions of cognitive psychology has exactly been the distinction between two stages in problem solving: *representation* (understanding the problem) and *solution* (searching the problem space). The distinction between these two phases, Mayer observes, is not always possible, although it suggests that difficulties noticed within problem solving activities may come from an inadequate representation.

In the end, the phase of representation of the problem is recognized as an essential and, at the same time, critical moment in problem solving. Researchers on the one hand underline that the representation process may be hindered by an excessively concise context (which rather favours the enactment of cognitive shortcuts, like those described by Sowder), on the other hand, claim that a story which is too rich can possibly "distract" the student (Gerofsky, 1996), and therefore hinder the solu-

tion process. The following examples, drawn from Italian research studies, seem to confirm this "distracting" effect:

1. Within a study carried out by researchers of the University of Modena on the probabilistic intuitions of 2-3 grade children the following problem was used (Zan, 2007):

Every time she goes to visit her grandchildren Elisa and Matteo, granny Adele carries a bag of fruit-candies with her and offers them to the kids, asking them to take the candies without looking into the bag.
Today she arrived with a bag containing 3 orange jellies and 2 lemon jellies.
If Matteo is the first to take a jelly, is it easier that he gets an orange or a lemon jelly?
Why?

Some children answer "orange", with the justifications: "Because he likes them better"; "Because he looked inside"; "If Matteo took the lemon jelly, only one was left and instead it is better to take the orange jelly".

2. The following problem was assigned to students from 7 to 13 grade (Ferrari, 2003):

In a house a Chinese pot was broken. In that moment 4 guys are in the house: Angelo, Bruna, Chiara and Daniele. When she gets back, the landlady wants to know who broke the pot and interrogates all four, one at a time. These are the single statements:
Angelo: "It was not Bruna";
Bruna: "It was a boy"; Chiara: "It was not Daniele";
Daniele: "It was not me".
Can you find out who is guilty? Careful: out of the 4 statements, 3 are true while 1 is false.
Who broke the Chinese pot? Explain how you found the answer.

These are some of the answers collected by Ferrari:
"It was Angelo, because he was not cleared by anyone"; "It was Chiara: nobody names her because they want to cover her"; "It was Daniele: he clears himself, therefore it was probably him".

This and other studies about word problems suggest that when the context is very poor (as it generally is the case for those quoted in the literature) students tend to forget about the story and infer the operations to be done straight from the text. Rather, when the context is very rich (as it is the case for granny Adele and the Chinese pot problems), they get confused with "extraneous factors" (Gerofsky, 1996), and get "lost" in the story. In this concern, Toom (1999, p. 38) remarks that the text of a word problem must be "purged of all irrelevant data".[3]

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3. Toom refers here to what he calls *non-real-world* problems (juxtaposed to realistic problems), which «purpose is to convey a *mathematical meaning*, that is the use of suitable concrete objects to represent or reify abstract mathematical notions» (Toom, 1999, p. 37).

3   Story problems as problems formulated by others

The points made above about the role of the representation of the problem situation in the solution process lead us to underline a characteristic of word problem solving, that in our opinion has a great relevance to understand students' behavior.

The presence of a text that characterizes a word problem is connected to a feature of word problems that makes them really different to *real life problems*: the one who is to solve the problem (the student) is other from the one who proposes it (either teacher or textbook). In other words, problems students work on at school are "proposed, and formulated, by another person" (Kilpatrick, 1987), and the (written) text is the usual way they are *posed*.

Some important implications for the process of understanding a problem follow from the fact that school problems are *hetero-posed* (i.e. posed by others).

The first implication is the presence of an explicit question, with the function of communicating to the one who solves it what has to be his/her goal: when the problem is self-posed the one who solves it does not need to make his/her own goal explicit to him/herself. If the problem is formulated by others, an explicit question is not necessary only if the context of the problem describes a situation perceived as problematic by the solver. For instance, if someone tells us the following story: "Maria's child is ill and tomorrow she should be in a meeting", we recognise the described situation as problematic[4], and there is no need for our interlocutor to pose a question. The implicit question is: "(In your opinion) how can Maria manage?".

The second implication concerns the particular goal that characterizes those who pose a word problem. Nesher (1980) stressing the stereotypical nature of arithmetic word problems, underlines the role played by the intentions of the author of the problem text:

> «for the sake of simplicity, the qualitative and quantitative considerations for a given REAL-PROB [real problem] have already been made by the author of the text. (…) He has in mind a mathematical operation, or a mathematical structure with whose applications in real life he would like the students to become acquainted. The author then chooses one of the real life contexts and imagines a situation (…) which will call for the application of the given mathematical structure (…). In order to simplify it for the student he then adds, in the most concise manner, all the qualitative and quantitative information needed for solving the problem, and arrives at a kind of SCH-PROB [school problem] which has all the stereotyped characteristics already described.»
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> (Nesher, 1980, p. 45)

For example, if the author has in mind the mathematical problem:

It takes a time t to cover a distance s with speed v_1. How much does it take to cover the same space with speed v_2?

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4. The problematic feature detected is not intrinsic to the situation, it is rather socio-culturally situated: we recognise it because the *story* of Maria somehow recalls our own similar experience, our lived life, our emotions … in the end our knowledge of the world. The same story told in different socio-cultural contexts (for examples to little children, or to people who have only experienced patriarchal families) might not be recognized as a problematic one.

he can embed it in a real life context such as:

> John takes 20 minutes to go from home to work, travelling at a 40 km/h speed.
> Today he is late and travels at a speed of 50 km/h. How long will he take?

Thus the goal of either the teacher or the author of the text is *internal* to mathematics. Nevertheless, as Cobb (1986) remarks:

> «(...) there is a gross mismatch between the goals that the teacher thinks he
> or she is getting for students and the goals that students actually seek to
> achieve. In other words, the teacher believes that the students are operating in
> a mathematical context when their overall goals are primarily social rather than
> mathematical in nature.»[5]
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> (Cobb, 1986, p. 8)

Cobb explicitly points to a "social" context, meaning that students' activity «is directed toward the goal of either bringing about or avoiding certain responses from the teacher» (ibidem, p. 8). More in general he claims that «the psychological context within which one gives a situation meaning can radically affect subsequent behavior» (ibidem, p. 2). In the case of a story problem the psychological context involved in the phase of representation requires «penetrating to the pragmatic deep structure» (Nesher, 1980, p. 46) of the story. Understanding the stories of people, of their reasons, intentions, feelings is linked to a form of thought that Bruner (1986) defines as "narrative", and that the scholar juxtaposes to "paradigmatic" or "logico-scientific" thought:

> «There are two modes of cognitive functioning, two modes of thought, each
> providing distinctive ways of ordering experience, of constructing reality. The
> two (though complementary) are irreducible to one another. (...) One mode, the
> paradigmatic or logico-scientific one, attempts to fulfill the ideal of a formal,
> mathematical system of description and explanation. It employs categorization
> or conceptualization and the operations by which categories are established,
> instantiated, idealized, and related one to the other to form a system... (...) The
> imaginative application of the narrative mode leads instead to good stories,
> gripping drama, believable (though not necessarily "true") historical accounts.
> It deals in human or human-like intention and action and the vicissitudes and
> consequences that mark their course. It strives to put its timeless miracles into
> the particulars of experience, and to locate the experience in time and place.»
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> (Bruner, 1986, p. 11-13)

The representation of the situation described in the word problem – the "story" – thus requires the student to get into a context (in the sense of Cobb) that we might call *narrative*. Then, on the representation of the situation, the solution process (and the answer with it) should be built, and logical thought plays a crucial role in this. Distinguishing between the two phases of *representation* and *solution* as well as the role played by narrative and logical thought in these phases leads us to distinguish

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5. This mismatch clearly emerges from the answers given by some children to the question "What is a problem to you?" (Zan, 2007): "An example of problem may be that of a mathematical problem I cannot solve." [Simone, grade 5].

between information relevant to representation (that we might call "narratively relevant"), and information relevant to the solution, that is to answer the question ("logically relevant"). The point here is that the data a child needs to represent the problem are not necessarily those he/she will need to use in the solution.

For example, in the Chinese pot problem, the fact that the broken object was a Chinese pot was relevant for the story and therefore for the representation process (if instead of a Chinese pot a simple glass had been broken, the story would not have made sense), even if not relevant for the solution.

The importance of narratively relevant information to solve the problem clearly emerged from a study carried out by D'Amore et al. (1996). Children from 4th to 8th grade had to re-formulate the following word problem: "Three workers take 6 hours to complete a certain job. How long will 2 workers take to complete the same job?"

All the children added information about *the reason why* the workers reduced from 3 to 2 (for example: "one got ill *and so* only 2 were left"). This piece of information was relevant to grasp the story, in particular its problematic nature and (therefore) its relationship with the question. In a later study I supervised for a first degree, some children explicitly commented upon the original text as follows: "I can't imagine the scene because I don't know what their job is", "I can't understand how to answer the question because the workers are initially three and then they become two, it is not explained very well".

The need of linking the two pieces of information ("three workers at a certain job" and "two workers at the *same* job") with a story also emerges from most drawings children made to represent the problem:

Among the information relevant for the solution a crucial role is played by the question: in order to solve the problem, a child must represent the situation but *also* understand the sense of the question. Therefore, the more the representation of the described situation evokes the question to the child, the more that representation will promote an understanding of the question, needed to get to the solution. Particularly meaningful from a narrative standpoint is therefore that piece of information that enables the child to grasp the problematic nature of the story and point out the link existing between the story itself and the posed question.

It might also happen that the pieces of information needed to solve the problem are not necessarily consistent from a narrative viewpoint, and if they are inconsistent, they will probably be ignored by those who read in a narrative mode.

For instance, in the "Granny Adele" problem the final question ("Is it easier to get an orange or a lemon jelly?") is not a realistic one from a narrative standpoint: it is an artificial question, with no meaningful links to the narrated story. Actually the reported answers suggest that the children either answered a different question[6], or simply completed the story. Similarly, in the "Chinese pot" problem – which follows the narrative plot of a "thriller" – the piece of information *Careful, though: out of the 4 statements, 3 are true whereas 1 is false* is not consistent from a narrative viewpoint (who may know?) and it is nevertheless a fundamental one to answer the question. In the end, in order for narrative thinking to *support*, through the process of representation of the story, logical thinking, which is necessary for the solution process, it is important that information needed for the solution be consistent from a narrative viewpoint, and that information needed for the representation be consistent from the logical viewpoint, in particular consistent with the posed question.

In actual fact the standard formulation of story problems generally pays little attention to these aspects.
When the context is extremely poor, there is not enough information to represent the story: we might even say that sometimes a *story* as such is missing, given that a crucial dimension of stories is that of time (Bruner, 1986). In particular the question does not follow in a *narrative* way from the context, it is rather an artificial question about the context. The role of the context is reduced to that of container of necessary (and generally sufficient) data to be able to answer the question.
Thus, not surprisingly, the student focuses on the question, while the context is read with relation to that question (in particular by selecting key-words and numerical data). A typical example of a standard formulation is the following:

> Carlo buys a workbook and two pens. He spends 2 €. One pen is 0,6 €.
> How much is the workbook?

The context is not problematic, and it doesn't suggest any "natural" question. The posed question ("How much is the workbook?") is an artificial question *about* (and not *from*) the context.

On the contrary, when the context is pretty rich, the richness of the story promotes the enactment of narrative thinking, needed to understand it. It is not trivial that this understanding should support the solution process, and particularly the understanding the sense of the question. If the posed question does not narratively fit with the narrated story, or the story itself embeds essential information from the logical point of view but inconsistent from the narrative point of view, narrative thinking enacted by the story will not support the student in solving the problem. It may even be an obstacle in the solution process, leading the child to answer a question that better fits with the narration, or rather to get lost in the "fictional wood[7]" we have built for him/her.

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6. Margaret Donaldson (1978) proposes the same interpretation, when she discusses the wrong answers given by children in the typical tests of Piaget. Donaldson criticizes the structure of those tests, in particular pointing to the fact that the question has little "meaning" in the experimental context. She makes the hypothesis that a child's failure to complete some typical tests (about point of view, about conservation, about sub-classes) might be ascribed to the fact that the child actually answers a different question, more consistent with the context. In actual fact, changes of the experimental context aimed at making the question "meaningful" bring about an increase of right answers.
7. The *fictional wood metaphor* is an idea of Umberto Eco (1994).

In our hypothesis this is what happens in the "Granny Adele" problem as well as in the "Chinese pot" one. In fact the answers we reported suggest that children have completed the story *narratively*, regardless of either the posed question or the given constraints: the story of a grandmother with her grandchildren, the "thriller" of the Chinese pot. In these cases a phenomenon described by Cobb seems to take place: the child works in a setting – the narrative one in this case – that differs to the logical-mathematical one, expected by the teacher. In this setting the answers reported are fully legitimated, and it does not make sense to talk about mistake or even "lack of logical thinking". But, in our hypothesis, this is also what happens in the "John problem" *(John takes 20 minutes to go from home to work, travelling at a 40 km/h speed. Today he is late and travels at a speed of 50 km/h. How long will he take?)*. Some high school students find this problem difficult, because "there are not enough data to answer". In actual fact in this case the context represents a problematic situation (John is late at work) which suggests a "natural" question: *"Will he arrive on time?"* And there are not enough data to answer this question!

4 Conclusions

The phase of understanding the problem is acknowledged to be at the same time a crucial and a critical moment of the solution process. Research on word problems, in particular, highlights that on the one hand the process of understanding may be hindered by a context that is too concise, on the other hand that an excessively rich story may "distract" children. Our remarks suggest a possible interpretation of this phenomenon, which requires further investigation: the difficulties highlighted in the two cases are due to a lack of consistency between the narratively relevant and logically relevant information, more than to the features of the context taken as such (concise/rich). In the case of a rich context lacking of this consistency, narrative thinking enacted by the context does not support logical thinking, needed to give the answer: a consequence of this might be that narrative thinking prevails and the child gets lost in a fictional wood. Conversely, in a "well formulated" problem the story described in the context supports, and does not hinder, the solution of the problem itself. The following is an example of a "good formulation (in this sense) of the "Carlo problem":

> Andrea must buy a workbook but cannot go to the stationer's. Thus he asks
> Carlo to buy it for him. Carlo though, besides the workbook for Andrea buys
> two pens, each costing 0,6 €, for himself.
> The overall cost is 2 €.
> When Carlo gives the workbook to Andrea, Andrea asks him: "How much do
> I owe you for my workbook?"
> How can Carlo know that?

Because of the purposes of the activities in the classroom with word problems, the teacher (or the author of the problem) pays most attention to the solution process, and therefore to the question and to the information needed to answer. The same attention though, is not paid to the phase of representation, and therefore to the information a child needs to represent the problem to him/herself: information concerning the "story" is often viewed as "irrelevant" details, source of confusion rath-

er than help. In other words, it is the logical structure of the problem that deserves the attention of those who pose the problem, whereas the narrative structure is not considered enough. Hence, what generally happens is that there is a "narrative rupture" in the text of the problem, i.e. the question and the information needed for the solution are not consistent from the point of view of the narrated story.

In the examples we examined, this narrative rupture occurred in most cases between the context and the question (Granny Adele, John's delay and Carlo problems), but the narrative rupture can also be located within the context itself (as in the Chinese pot problem).

The analysis proposed in this paper refers to word problems characterized by a *story*. Our remarks suggest that research about word problems should appropriately consider story problems as "particular" word problems, whose specific features deserve researchers' attention. Further investigation about *story problems* is needed: in particular, it is possible that the process of understanding may be favoured by the presence of a story. For instance, the time-dimension which characterizes a story is also the main difference between *static* and *dynamic* problems, where a change happens, and research pointed out that dynamic problems turn out to be easier for children than static ones (Nesher, 1980).

Another implication of our remarks is that two logically equivalent problems might be very different from a narrative viewpoint. Being able to recognize the same mathematical structure in different story problems is an important skill in mathematics, which involves logical thinking and which cannot be viewed as a pre-requisite. It is rather an end-point of mathematical education, requiring time and attention to the critical points we stressed. The link between contexts and modes of thought (logical/narrative) on the one hand, and between contexts and goals, on the other, as underlined by Cobb (1986), points out another important objective for mathematics education, at a meta cognitive level: educating students to recognize contexts in which logical thinking better fits with their purposes. But, again with Cobb, an individual's goals are in turn linked to his/her beliefs, that «can be thought of as assumptions about the nature of reality that underlie goal-oriented activity» (Cobb, 1986, p. 4). This link highlights the role of the beliefs students build up by interpreting their own experience.

One last remark. Even though a mathematics teacher's task is to develop logical thinking, in my opinion narrative thinking should not be viewed as an obstacle to logical thinking, or anyway as a lack of rational thinking. It might work in contexts where logical thinking fails. For example, narrative thinking may let a logically absurd problem make sense. In the problem known as "the age of the captain" *(There are 26 sheep and 10 goats on a ship. How old is the captain?)* some children answer summing up the numbers found in the text and justify this with arguments like Perhaps the captain got an animal as a gift for each birthday (IREM de Grenoble, 1980), i.e. they build up a *story* thanks to which the answer (and thus the question) makes sense. Hence, given the problem *In a meadow there are 20 sheep, 7 goats and 2 dogs. How old is the shepherd?* (logically, but not narratively equivalent to the previous one) a child answers:

> «My particular reasoning path was: if the shepherd has two dogs for few animals, perhaps he needs one of the two dogs because he is blind. Hence I deduce that he might be about 70-76 years old.»

Does this answer really show "suspension of sense-making"?

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